Energy Loss of Slow and Fast Ion Beams in an Ordered and a Disordered Plasma

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Abstract

In this framework which involves the linear response formalism and by using a number- conserving relaxation-time approximation to include an ordered and a disordered degenerate electron plasma we have shown, through analytical and numerical results, that are stopping power, friction coefficient, and effective electron number, which are produced by the interaction of energetic point-like or extended projectile ions with metals, are significantly affected by damping, ionization, electron density, low and high projectile velocity.

الخلاصة

في إطار البحث هذا والذي يتضمن صيغة الاستجابة الخطية وباستخدام إعداد تقريب حفظ زمن الاسترخاء لتتضمن بلازما الكترونات متحللة بشكل منتظم وآخر غير منتظم تم إثبات, ومن خلال النتائج التحليلية والعددية, بان قدرة الإيقاف ومعامل الاحتكاك والعدد الالكتروني الفعال الناتجة من تفاعل المقذوفات الأيونية شبه النقطية أو الممتدة تتأثر بشكل مهم بالخمود والتأين والكثافة الالكترونية والسرع الواطئة والعالية لتلك المقذوفات.

1. Introduction

The energy loss of a charged particle moving in a degenerate electron gas (DEG) has greatly interesting. This is a topic acquired its importance through the presented quantitative understanding, for instance, of beam target interaction in the contexts of particle driven fusion [1] and the implantation experiments such as the modification of metal surfaces using ion beams. Following the pioneering works of Lindhard [2] and Lindhard and Winther [3], a lot of calculations were done within the framework of linear-response function (see e.g. [4–9] for reviews). The main part of these calculations was based on the dielectric function in the random-phase approximation (RPA) which is valid mostly in the high-density regime $r_s < 1$ of a target electron gas. However, the RPA theory by itself cannot provide accurate values of the stopping power (SP) for real solids such as metals and semiconductors where correlation (beyond RPA) and/or damping effects (e.g. due to electron–impurity collisions) play an important role. One particularly simple scheme to include electron–impurity collisions in a disordered electron plasma is provided by the number-conserving relaxation-time approximation (RTA) as suggested first by Mermin [10] and then by Das [11] in RPA and for a given electron–impurity collision frequency γ, resulting in the dielectric function $\varepsilon(k,\omega,\gamma)$. We have used this scheme to calculate the energy loss of point-like and extended energetic ion beams in an ordered and a disordered plasma as well as friction coefficient and effective electron number to show the influence of γ.

2. Theoretical model

Considering an extended ion, with the charge Ze of the point-like nucleus and N bound electrons, moving with velocity V in a target electron gas characterized by the dielectric function $\varepsilon(k,\omega,\gamma)$, the SP is given by [4–9]

$$S(\lambda)=\frac{16Z^2\Sigma_0}{\pi^3\chi^4\lambda^2}\int_0^\infty G^2(z)zdz\int_0^\lambda \operatorname{Im}\frac{-1}{\varepsilon(z,u,\gamma)}udu$$
$$=\frac{8\Sigma_0 Z^2}{3\pi^2\chi^2\lambda^2}L(\lambda). \qquad (1)$$

In Eq.(1) $z=k/2k_F$ and $u=\omega/kv_F$ where v_F and k_F are the Fermi velocity and wave number of the target electrons. $L(\lambda)$is the stopping number. $\lambda = V/v_F$, $\Sigma_0=e^2/2a_0^2$ = 2.566 GeV/cm (a_0 is the Bohr radius), $\chi^2=1/\pi k_f a_0$, $G(z)=1-z^{-1}\rho(z)$, where $\rho(z)$ is the Fourier transform of the spatial distribution of bound electrons in the ion. The density distribution of bound electrons can be found through the Brandt–Kitagawa variation statistical approximation [9] with an ion effective radius Λ. In this approximation the spatial distribution of bound electrons is given by $\rho_e(r)=\left(N/4\pi\Lambda^2\right)\left(e^{-r/\Lambda}/r\right)$ [9]. Then introducing the notations $q=N/Z$, $q_1=1-q$, and $\alpha=1/2k_F\Lambda$, we have

$$G(Z)=\frac{Z^2+\alpha^2 q_1}{z^2+\alpha^2}, \quad \Lambda=\frac{0.48a_0}{z^{1/3}}\frac{q^{2/3}}{1-q/7} \qquad (2)$$

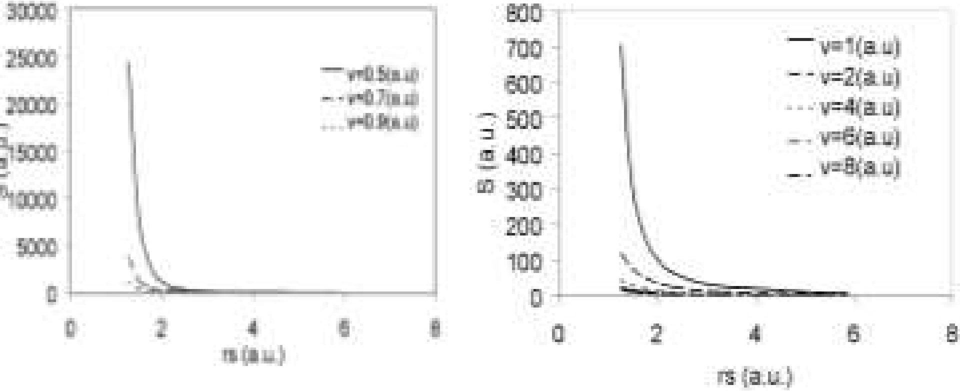
The linear response function $\varepsilon(z,u,\gamma)$ includes disorder in the RPA [8,9,12], γ being a measure of disorder. The stopping number which is found [12]

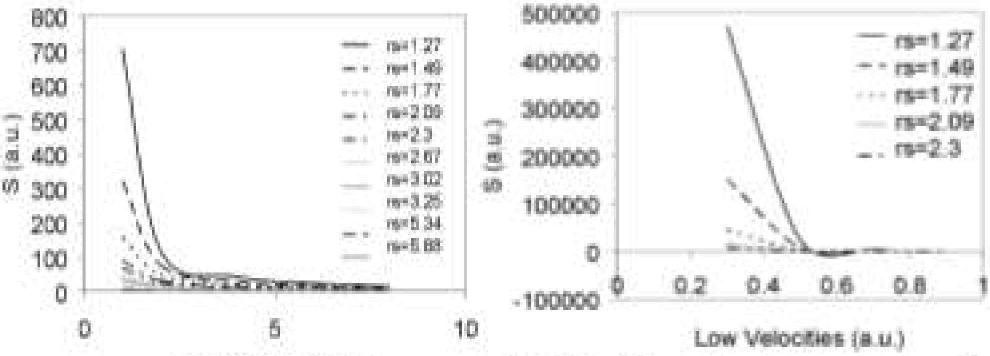
$$L(\lambda)=q_1^2\ln\left[\frac{2mV^2}{\hbar\omega(\Gamma)}\sigma\left(\frac{V}{V_0}\right)^{\sigma_1}\right]-\frac{C_1}{\lambda}-\frac{C_2}{\lambda^2}-\frac{C_3}{\lambda^3}-\frac{C_4}{\lambda^4}-\cdots\cdots, \qquad (3)$$

with $\Gamma=\hbar\gamma/4E_F, E_F$ being the Fermi energy .Here $\sigma_1=q_1^{-2}-1, \sigma=e^{-v^2/2v_0^2}(2\Lambda/a_0)^{\sigma_1}$, $Q(\eta)=(\eta/\eta_1)$ arcsin η, $\eta_1=\sqrt{1-\eta^2}$, $\omega(\Gamma)=\omega_p e^{Q(\eta)}$, $v_0=e^2/\hbar$ .The function $Q(\eta)$ depends on the damping parameter C through the relation $\eta=\Gamma\sqrt{3}/2\chi$ and has been obtained under the assumption $\eta<1(or\gamma<2\omega_p)$. For small damping $(\Gamma\to 0)$ the function $Q(\eta)$ vanishes and increases with damping parameter Γ and at $\eta=1(\gamma=2\omega_p)$, $Q(\eta)=1$ .The coefficients $C_i(\Gamma,q)(i=1,2,\cdots\cdots)$ in Eq.(3) depend on damping and the ionization factor q.

3. Energy loss of ion beam in an ordered plasma (γ=0).

Under this subject it has been studied the energy loss of firstly, point-like projectile of Oxygen ion where the ionic parameter (q) equals zero through different electronic densities which are represented by the values of density parameters (r_s) for different metals at low and high velocities of that projectile (see Fig 1) in this case the most important matter is the usage of the conservative constants[12] $C_1=C_3=0$, $C_2=3/5, C_4=3/14+x^2/3$ to calculate the stopping number $L(\lambda)$ as obtained by Lindhared [2].





(a)
(b)
(c)
(d)

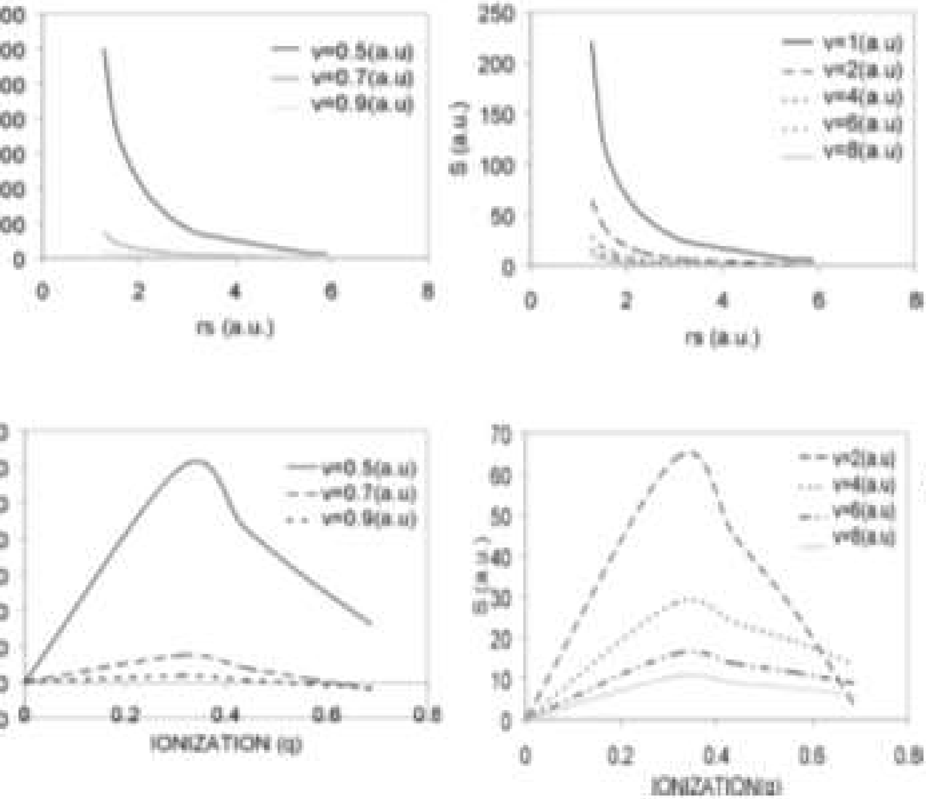
Fig(1): The stopping power of point-like projectile (q=0) versus density parameters for different materials at (a) low , (b) high velocities respectively , (c) , and (d) versus high and low velocities for different materials.

Secondly, under the same subject this research deals with the case of extended ionic projectile i.e. the ionic parameter (q) unequals zero where the conservative constants[12] are $C_1=C_3=0$,

$$C_2=\frac{3(1+q_1^2)}{10}+\frac{qq_1\chi^2}{3\alpha^2}-q\alpha^2,$$

$$C_4=\frac{1+q_1^2}{2}\left(\frac{3}{4}+\frac{\chi^2}{3}\right)+\frac{qq_1\chi^2}{5\alpha^2}-\frac{\chi^4 q(2-3q)}{36\alpha^4}+\frac{\alpha^4 q(2+q)}{4}+q\alpha^2 . \tag{4}$$

The energy loss of the extended ionic projectile has been investigated for the same conditions which are stated above as well as the effect of ionization parameter (see Fig.2).



(b)

(c) (d)

Fig2: (a), and (b) Stopping power of extended ion (q= 0.313) versus density parameters of different materials (an ordered plasma) at low and high velocities respectively ,(c) and (d) stopping power versus ionization coefficients in an ordered plasma at low and high velocities respectively .

4. Energy loss of ion beam in disordered plasma ($\gamma \neq 0$).

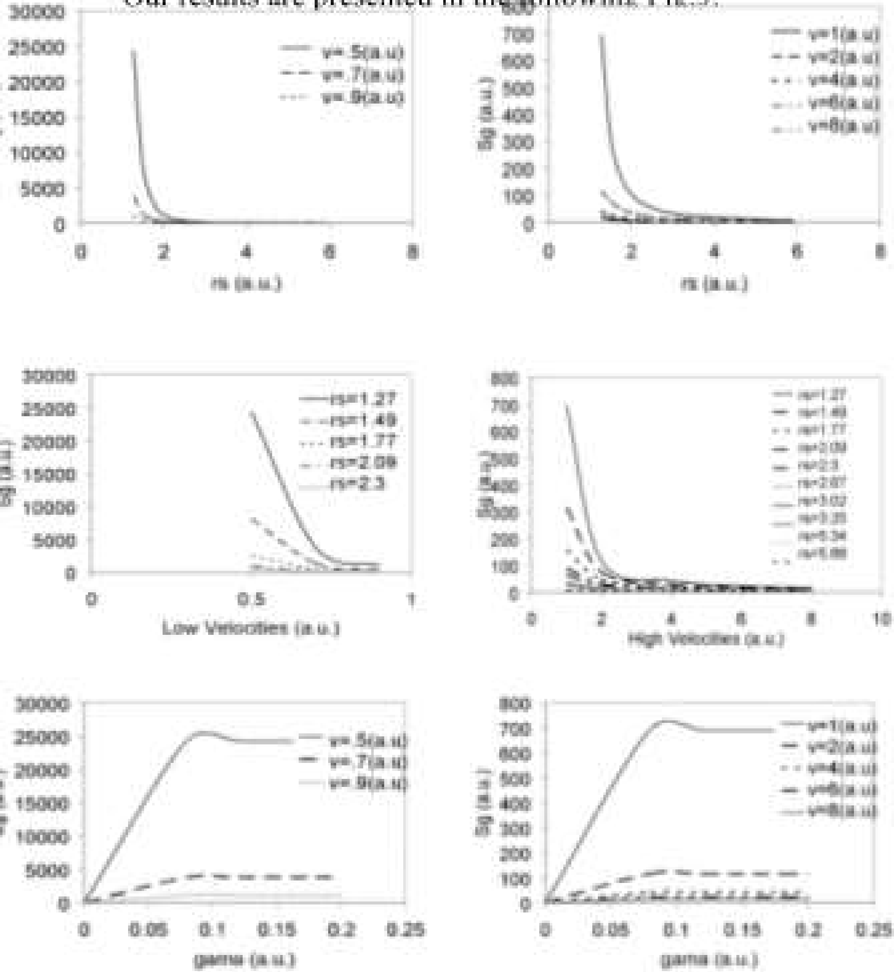
Study of stopping power has been achieved for point–like and extended ionic projectile with low and high velocity . This investigation include the variation effect of density parameters by using several values of them in calculation the stopping power and then to study their influence .

Within our presented conditions and to calculate the stopping power of the point–like projectile by different disordered targets of plasma which contain foreign atoms to produce collisions with electrons by frequency of γ which represents the reciprocal of mean free time, here we have to use the following conservative constants[12] in Eq.3, where $C_1=C_3=0$,

$$C_2 = \frac{3}{5} - \frac{13\Gamma^4}{5\chi^4},$$

$$C_4 = \frac{3}{14} + \frac{\chi^2}{3} + \frac{2\Gamma^{-2}}{7}\left[\frac{27\Gamma^2}{5\chi^4}\left(\frac{\Gamma^4}{5\chi^4} - 3\right) + \frac{14\Gamma^4}{\chi^4}\left(1 + \frac{24}{25\chi^2}\right) - \frac{119}{12}\right] \quad (5)$$

Our results are presented in the following Fig.3:

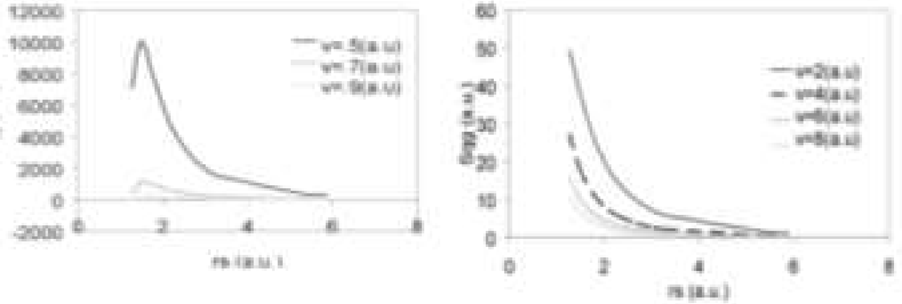


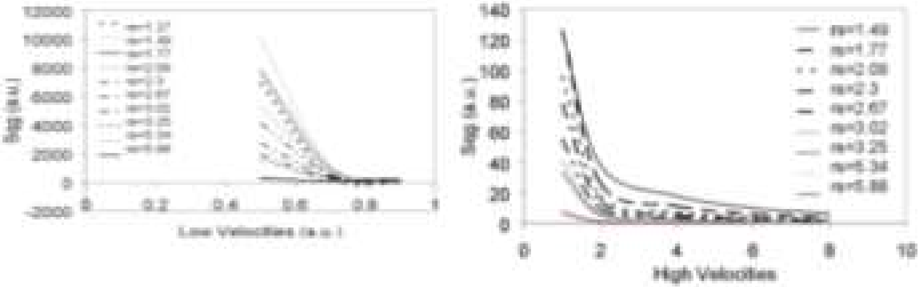
(b)

(e) (f)

Fig.3:(a), and (b) stopping power of point–like ion versus density parameters of different materials (a disordered plasma) at low and high velocities respectively. (c) ,(d) stopping power of point –like ion versus low and high velocities respectively for different materials (a disordered plasma). (e) and (f) stopping power of point –like ion versus gama parameter at low and high velocities respectively.

The calculations of the stopping power of extended ionic projectile in a disordered target of plasma have done and the results as the following Fig.4:





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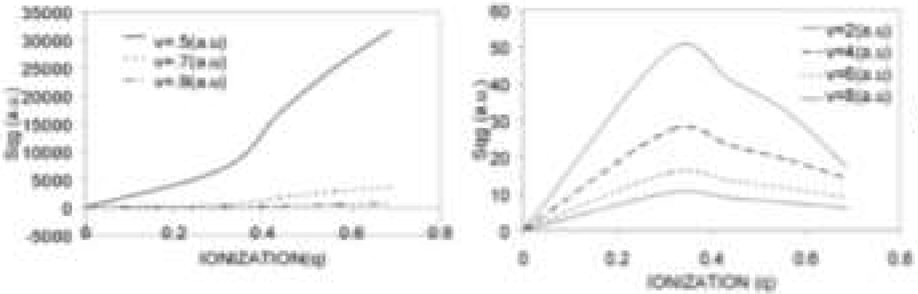
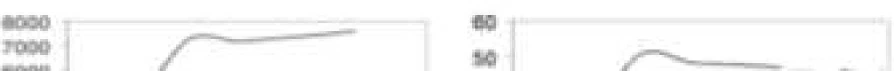


(d)



(e) (f)

(g) (h)

Fig 4: (a) , and (b) stopping power of extended ion versus density parameters of different materials (a disordered plasma) at low and high velocities respectively. (c) ,(d) stopping power of extended ion versus low and high velocities respectively for different materials (a disordered plasma). (e) and (f) stopping power of extended ion versus ionization parameter at low and high velocities respectively. (g) and (h) stopping power of extended ion versus damping parameter at low and high velocities respectively.

here we have used the following conservative constants [13] in Eq.3,

$$C_1 = \frac{\Gamma q}{\alpha}\left(3 - \frac{35}{16}q\right), \quad (6)$$

$$C_2 = \frac{3(1+q_1^2)}{10} + \frac{2qq_1}{\alpha^2}\left(\frac{\chi^2}{3} - \Gamma^{-2}\right) - \frac{12q_1^2\Gamma^{-4}}{5\chi^4} \quad (7)$$

$$C_3 = \frac{2\Gamma q}{3}\left\{\frac{9}{10\alpha}\left(3 - \frac{35q}{16}\right) - \frac{3\alpha}{2}\left(1 - \frac{5q}{10}\right) - \frac{5(3\Gamma^{-2} - 2\chi^2)}{6\chi^3}\left(\frac{21q}{16} - 1\right) - \frac{4}{3\pi\alpha^2}\left[\frac{5q-3}{\alpha^2} + \frac{7(16-33q)}{24\pi\alpha^3} + 2q_1\right]\right\}, \quad (8)$$

$$C_4 = \frac{1+q_1^2}{2}\left(\frac{3}{14} + \frac{\chi^2}{3}\right) + \frac{2q_1^2\Gamma^{-2}}{7}\left[\frac{27\Gamma^{-2}}{5\chi^4}\left(\frac{\Gamma^{-6}}{5\chi^4} - 3\right) + \frac{14\Gamma^{-4}}{\chi^4}\left(1 + \frac{24}{25\chi^2}\right) - \frac{119}{12}\right] - \frac{3\Gamma^{-2}q(2-q)}{2} + \frac{q(5q-3)}{2\alpha^4}\left(\frac{\chi^4}{9} + \Gamma^{-4} - \Gamma^{-2}\chi^2\right) - \frac{9\alpha^4}{2} + \frac{2qq_1}{\alpha^2}\left[\frac{\chi^2}{5} + \frac{33\Gamma^{-2}}{20} - \right.$$

(9)

5.Results and discussion

This paper investigates the behavior of ionic projectile of Oxygen within several plasma states of different targets. The variation of these states come from firstly, kind of target plasma is ordered which means the frequency (γ) of electron –foreign atom collision equals zero. Secondly, if the target is a disordered, (γ) unequals zero, and with respect to the ionic projectiles are classified to point- like and extended projectile where ionization parameter (q) equals and unequals zero respectively. Parameter (q) can be calculated from the ratio of the number of bounded electron of the projectile (N) to its atomic number (Z), where for instance O^{+4}, N=12and Z=16 this leads to q=3/4 and so on.

We have used in this paper ten values of (q). The results of Fig.1 and Fig.2 illustrate changes of stopping power of point-like (S) and extended (S_q) projectile versus parameter of density (r_s) or Wigner Sietz radius for different metals which represents the distance between electrons of a target where $r_s = \left(\frac{3}{4\pi a^3 n}\right)^{1/3}$ [12],a is Boher radius =0.529 $\mathring{A}$ and n is the electron density of a matter. In general the cooling of ionic projectile by electrons of a matter is occurred by the direct interaction of the ion with the target electrons, but this matter demands to know the collision diameter bo [5] , where $b_0 = \frac{|Z|}{4\pi\varepsilon_o \mu V_r^2} e^2$ as well as the wavelength $\lambda_r = \frac{\hbar}{\mu V_r}$ related to the relative motion with reduced mass (μ) and velocity (V_r), hence we get Coulomb or Bloch –parameter $\eta_r = \frac{b_o}{\lambda_r} = \frac{|Z| e^2}{4\pi\varepsilon_o \hbar V_r}$ [5], the averaged relative velocity

of ion $\langle V_r \rangle = \left(\langle V_e \rangle^2 + V^2\right)^{1/2}$, where $\langle V_e \rangle$ is the averaged electron velocity where $\langle V_e \rangle = V_F(1+\frac{\theta}{2})^{1/2}$, where $\theta\langle 1$ for degenenerate electron and $V_F = \frac{1.919}{r_s}$ [5,14].
The presented relationships explain in general the reducing of stopping power versus the increasing of r_s which means reducing in the electron velocity or V_F and then Vr where b_o (collision diameter) related conversely with Vr. The same reason when V increases. Hence the rising of collision diameter makes the ionic projectile interacts with greatest number of electrons. This meaning is satisfied either at low projectile velocity to be low relative velocity or when increasing r_s leads to low Fermi velocity V_F, her we have to recall the electrons density proportion conversely with Wigner Seitz radius, and this is why the lowering of stopping with increasing of r_s (see Fig.1, Fig.2 , Fig.3, and Fig.4).

Actually there is a fact we must recall it that the energy loss of a projectile transfers from it to the electrons of target as numbers of quantum of energy represented by angular frequencies ω_P (plasmons) where $\omega_P = \sqrt{\frac{3}{r_s^3}}$, which means the size of plasmon affected conversely with Wigner Seitz radius [5].

It is very important to observe in Fig. 2c and d the change of the stopping power against ionization factors at low and high velocity respectively and in spite of our knowledge with the influence of the velocity on the stopping we find an optimum value of ionization factor approximately equals 0.3 ($r_s = 1.27$) at which projectile loses higher energy. This behavior produced by the logarithmic term depending on both functions of σ and σ_1 (see Eq.3).

In the case of a disordered plasma target ($\gamma \neq 0$), although the general behavior is similar to the case of an ordered plasma target Fig.3 and Fig.4 but we must state that the damping is more effective in the case of low velocity projectile than high velocity one.
In the beginning of ion penetrating a metal there would be interaction (produces excitation) between its charge and electron sea of matter [15], so the presenting of excitation energy $\hbar\varpi$ in the denominator of logarithmic term of stopping number reduces in general the stopping power if it is compared with that of an ordered plasma. But with the same disordered plasma damping can give rise in an observable effect and this agree with the result of Amal K. Das and Hrachya B. Nersisyan [12],this leads to decreasing in the stopping time with increasing of γ [16](see e and f Fig.3).

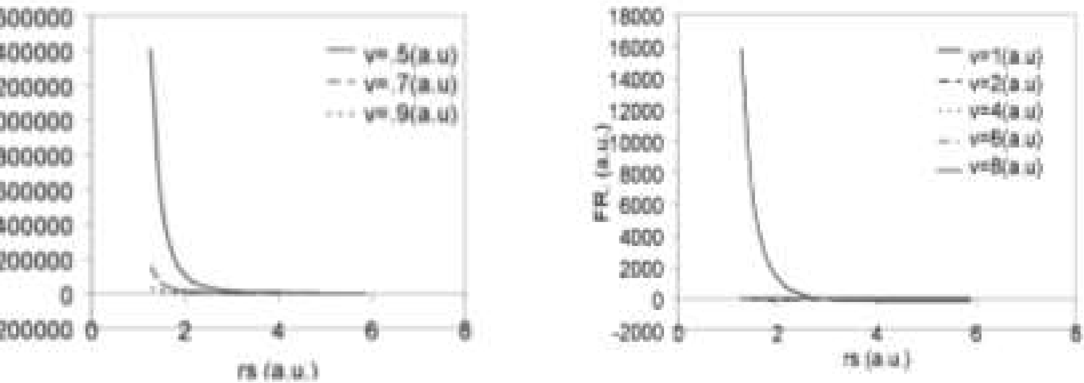
To examine the ionic stopping power for Oxygen closely, the subject of coefficient of friction has been investigated for both point-like and extended projectile. Friction property in the point of view quantum dynamic as it allows for localization, prevents back scattering, and is assented in the description of multistage transfer [17]. The latter coefficient is represented by the first derivative of stopping power, Eq.(1), with respect to the ionic projectile velocity [12], so we get generally

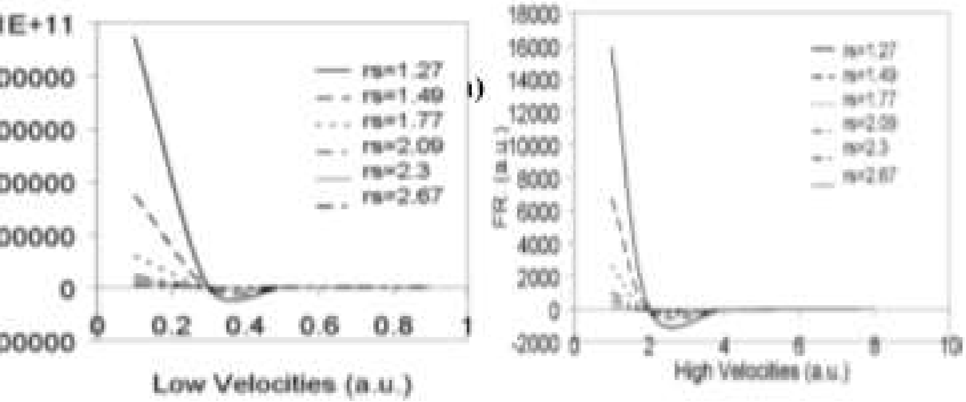
$$\frac{dS}{d\lambda}=\frac{8\Sigma_o Z^2}{3\pi^3\chi^2\lambda^3}\left[q_i^2\left\{\ln\left(\frac{2mV_p^{\,2}}{\hbar\varpi}\sigma\left(\frac{V_p}{V_o}\right)^{\sigma_1}\lambda^{2-\sigma_1}\right)^{-2}+(2+\sigma_1)\right\}+\frac{3C_1}{\lambda}+\frac{4C_2}{\lambda^2}+\frac{5C_3}{\lambda^3}+\frac{6C_4}{\lambda^4}\right]$$

(10)

The coefficient of friction for both project states in an ordered and a disordered plasma is calculated according to the conditions which limits values of $C_1, C_2, C_3, C_4, \ldots\ldots$ in above articles 3 and 4. The relation of stopping power in Eq.(1) proportion to the coefficient of friction ,Eq.10 [5] .

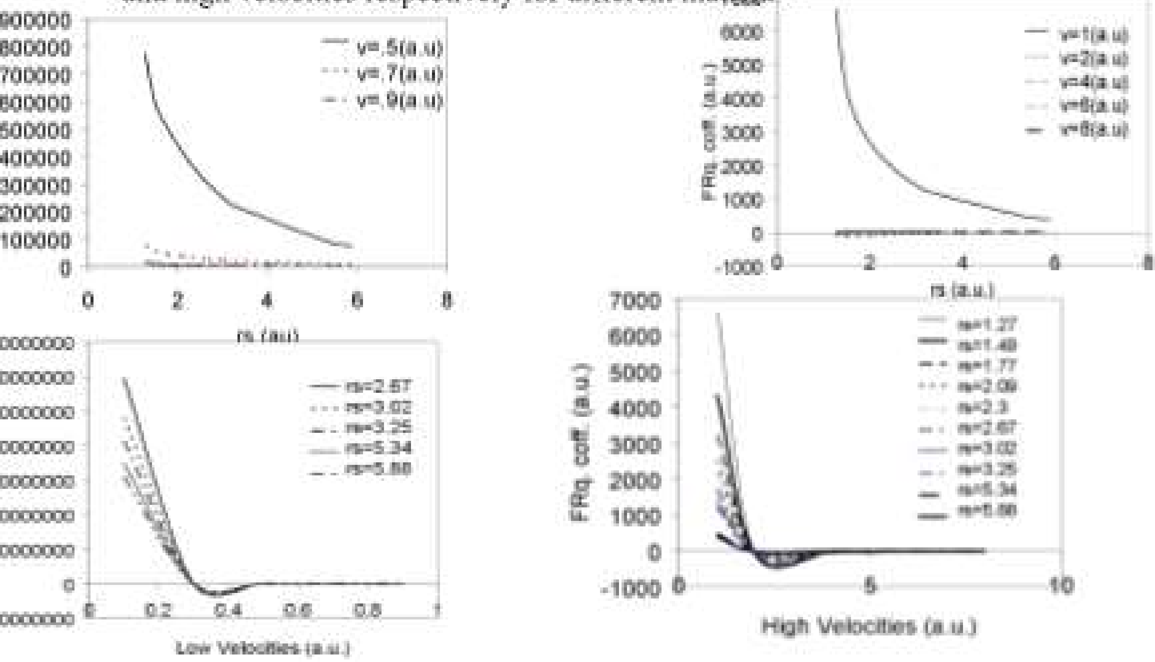
From Fig. 5 it can be seen the analogy of coefficient of friction for point –like projectile of oxygen impact a target of ordered plasma of different materials with the values of the stopping power of the same state . But this is not true for the other figures , Fig.6 ,Fig.7,and Fig.8, the latter figures includes negative values caused by the influence of the ionization and the damping $(\hbar\varpi)$ in logarithmic term .The functions σ and σ_1 depends on (q), then these graphs in general contribute in expressing the behavior of ionic SP and how to be affected by the ionization and the nature of the target plasma and its damping and electron density.



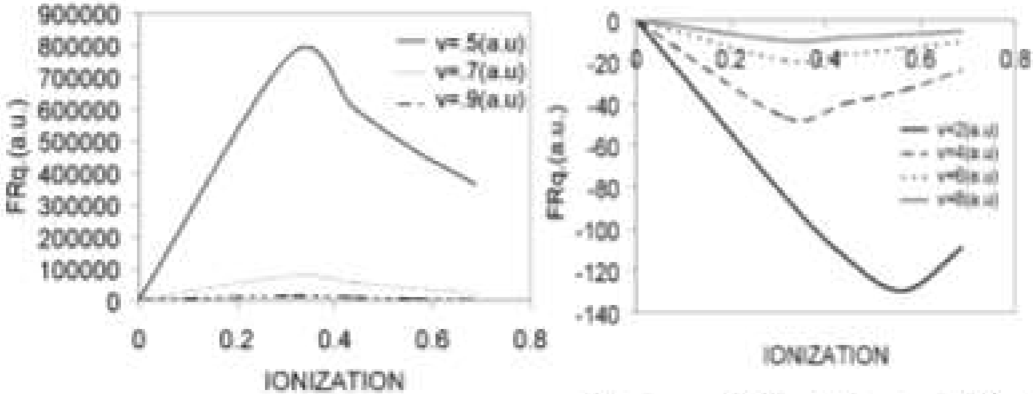


(b)

Fig(5): Friction coefficient of point-like projectile (q=0) versus density parameters for different materials at (a) low ,(b) high velocities respectively , (c) , and (d) versus low and high velocities respectively for different materials

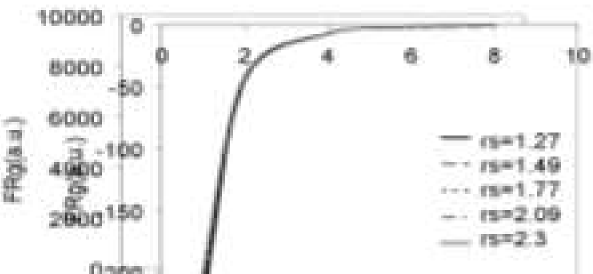


(c) (d)



(e)
(f)

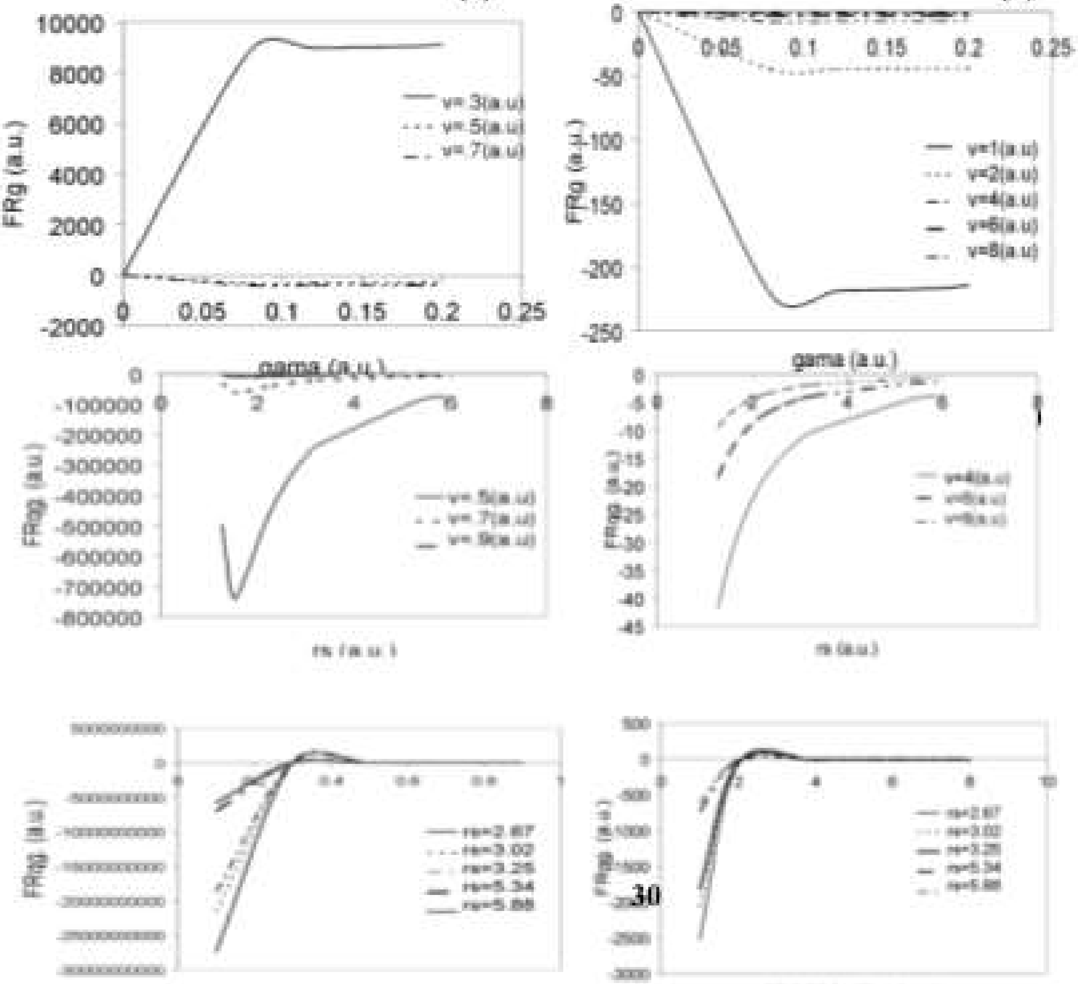
Fig6: (a), and (b) Friction coefficient of extended ion (q= 0.313) versus density parameters of different materials (an ordered plasma) at low and high velocities respectively ,(c) and (d) versus low and high velocities respectively for different materials, (e) and (f) versus ionization coefficients in an ordered plasma at low and high velocities respectively .



(a) (b)

Fig7: (a), and (b) Friction coefficient of point-like (q= 0) versus low and high velocities respectively for several density parameters of different materials (disordered plasma) at,(c) and (d) versus damping parameters at low and high velocities respectively.

(a) (b)



(g) **(h)**

Fig8: (a), and (b) Friction coefficient of extended ion (q= 0.313) versus density parameters of different materials (disordered plasma) at low and high velocities respectively ,(c) and (d) versus low and high velocities respectively for different materials, (e) and (f) versus ionization coefficients in disordered plasma at low and high velocities respectively, (g) and (h) versus gama parameters at low and high velocities respectively.

Now there is an other indictor explain the behavior of oxygen projectiles and their stopping power within plasma of different materials . This is so-called effective Number of electron (Neff..) it has been given by the following relationship [18].

$$Neff. = \frac{1}{2\pi^2 n}\int_0^E \mathrm{Im}\left[\frac{-1}{\varepsilon(k,\omega,\gamma)}\right]\omega d\omega$$

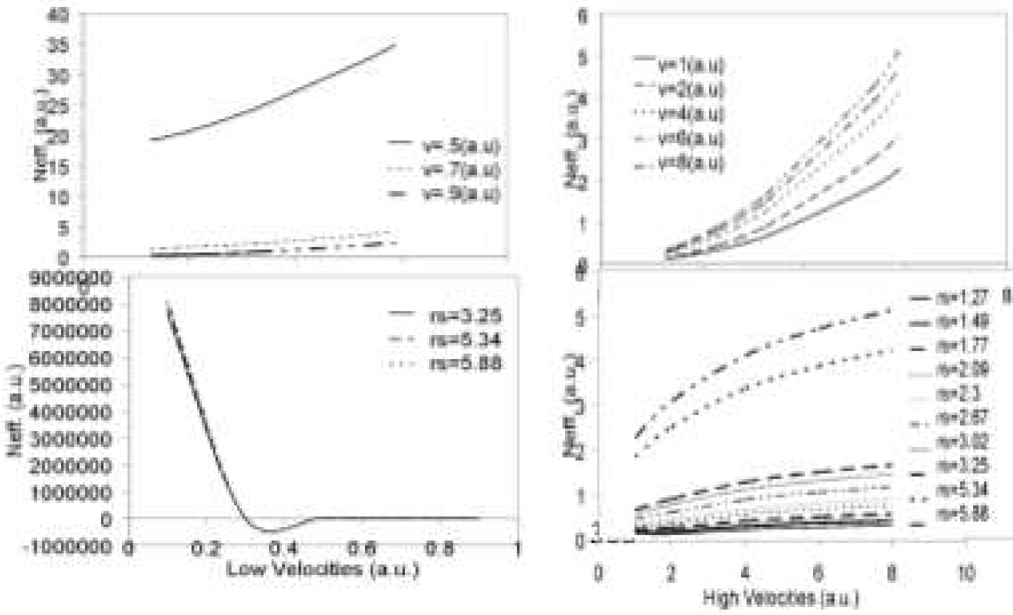
, where n is the electron density, E is the particle energy. The loss function $\left(\mathrm{Im}\left[\frac{-1}{\varepsilon(k,\omega,\gamma)}\right]\right)$ is derived in term of stopping number of (RTA)Eq.(3) by using Eq.(1) we get :

$\int_0^{\lambda} \mathrm{Im}\left[\frac{-1}{\varepsilon(k,\omega,\gamma)}\right]\omega d\omega = \frac{v_F^2}{3Int.(z)}L(\lambda)$, where *Int.(z)* is the result of numerical integral for $\int_0^{\infty} G^2(z) z dz$, see Eq.(1).

Hence we get

$Neff. = \frac{r_s^3 V_F^2}{8\pi Int.(z)} L(E)$, $Neff.$ is the effective No. of electrons, it can be calculated in case of an ordered plasma of different materials by substituting the conservative numbers which are referred in article (3) for both kinds of point-like and extended projectile. In addition this calculation can be used for the other kind of a disordered plasma of different material by substituting the conservative numbers of article (4) into stopping number, Eq.(3).

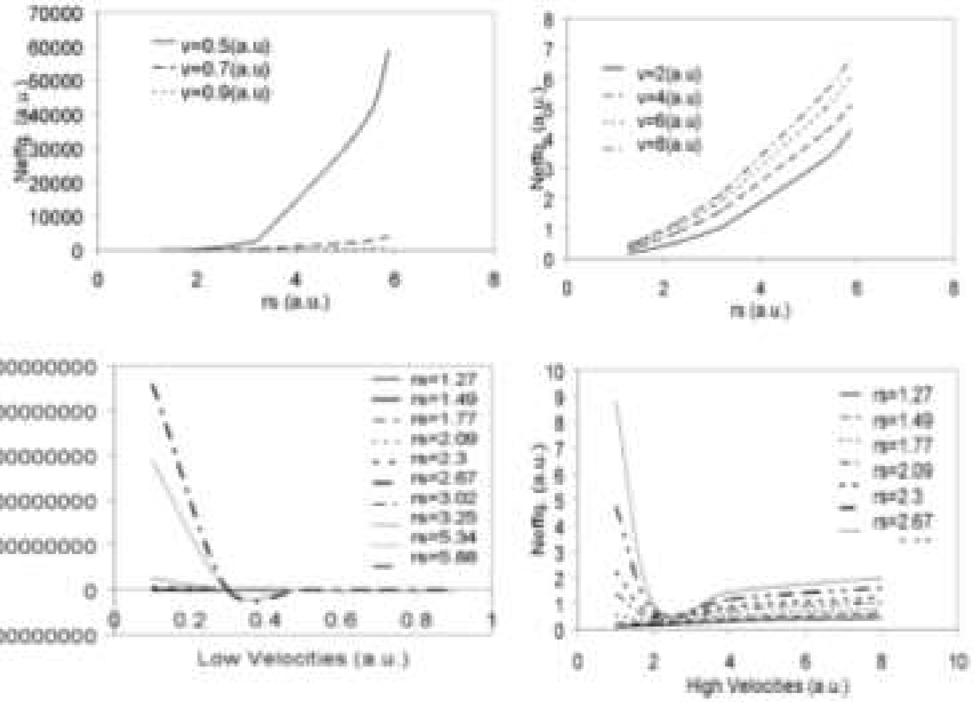
In general Fig.9, Fig.10 Fig.11,and Fig.12, illustrate that the increasing of Wigner Seitz radius means increasing the effective number of electrons of the targets and as a natural result this leads to the stopping of the projectile but we have to assign that in the case of low velocity the effective number of electrons being more than that of the case of high velocity projectile. Also we can see the medium velocity ($V = 1a.u.$) of the ionic projectile generally excites the lower effective number of electron. Where in this case the ion velocity approaches to the Fermi velocity and in spite of this state causes high energy transfer (as near as the resonance state) but the interaction spectrum being sharper than any state of lower or higher velocity with respect to the effective number of electrons. The contribution of ionization factor (appearing the influence of the ratio of screening length to the ion radius $\alpha = 1/2k_F\Lambda$)[12] is increasing the effective number of electrons see (e) and (f) in each of Fig.10 and Fig.12, as well as it can be seen in general how the effective electron number are affected by increasing the damping parameters (γ) and this appears obviously in each of Fig.11 and Fig.12.



(a)
(b)

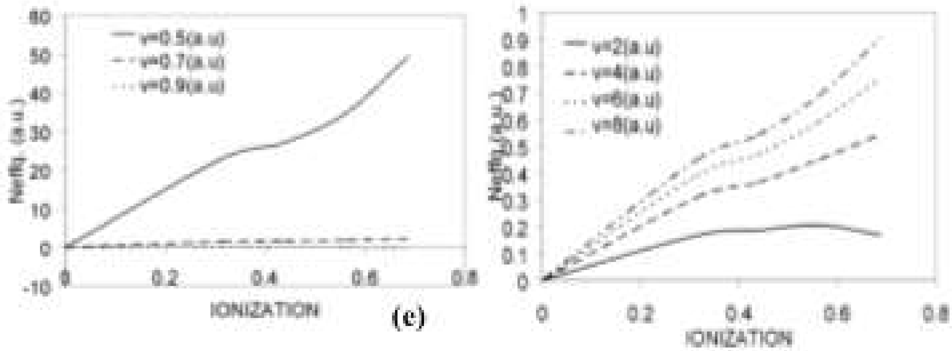
(c)
(d)

Fig9: (a), and (b) Effective electron number of point-like projectile (q= 0) versus several density parameters of different materials (ordered plasma) at low and high velocities respectively ,(c) and (d) versus low and high velocities respectively for different ordered target plasma.



(b)

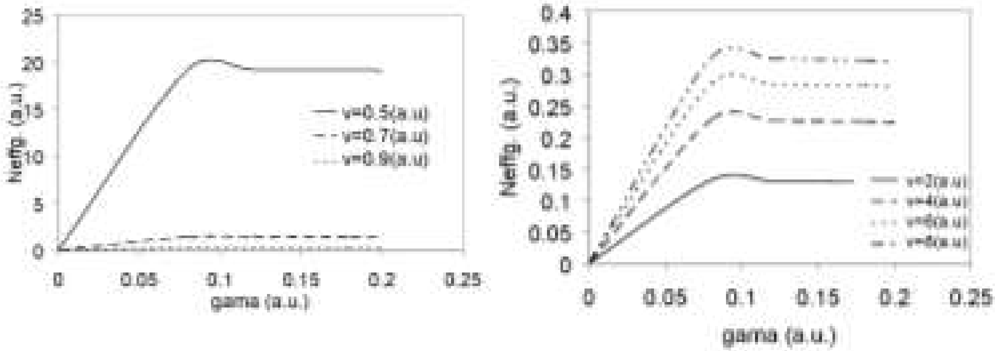
(d)



(e) (f)

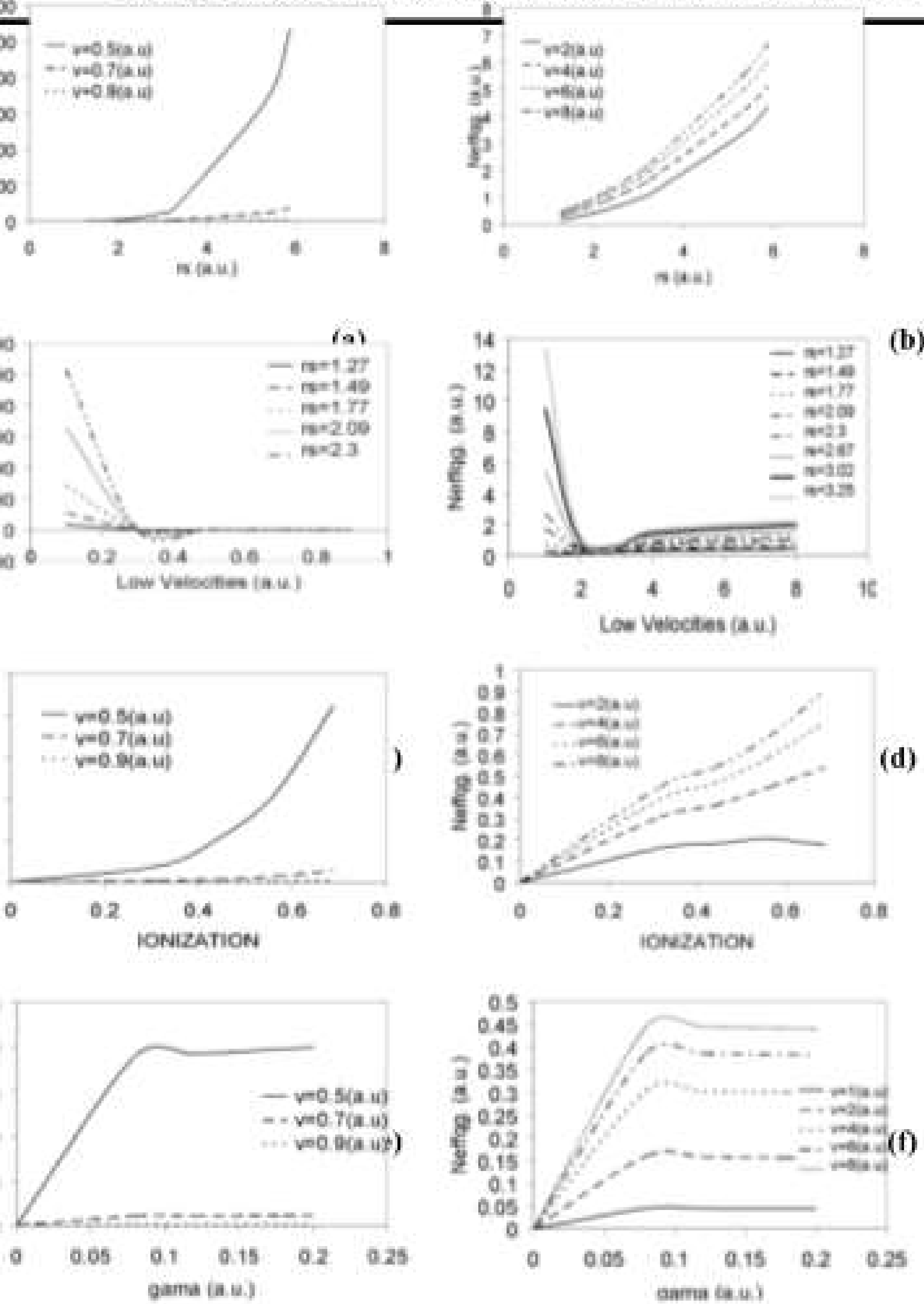
Fig10: (a), and (b) Effective electron number of extended ion (q= 0.313) versus density parameters of different materials (an ordered plasma) at low and high velocities respectively ,(c) and (d) versus low and high velocities respectively for different materials, (e) and (f) versus ionization coefficients in an ordered plasma at low and high velocities respectively .

(a) (b)



(c)
(d)

Fig11: (a), and (b) Effective electron number of extended ion (q= 0.313) versus low and high velocities respectively for several density parameters of different materials (disordered plasma) at,(c) and (d) versus damping parameters at low and high velocities respectively.



(g) (h)

Fig12: (a), and (b) Effective electron number of extended ion (q= 0.313) versus density parameters of different targets disordered plasma at low and high velocities

respectively ,(c) and (d) versus low and high velocities respectively for different materials, (e) and (f) versus
ionization coefficients in disordered plasma at low and high velocities respectively, (g) and (h) versus gama parameters at low and high velocities respectively.

6. Conclusion

- Within an ordered plasma in general energy loss, friction coefficient, and effective electron number for low projectile velocity are higher than that of the state of high velocity, also the point-like projectile loses higher energy than an extended projectile because of greater friction . The extended projectile is affected by ionization parameter especially with optimum value nears to 0.3.
- For a disordered plasma each of stopping power , Friction coefficient , and effective electron number have lower values than that of ordered plasma where ionization parameter play an effective role in reducing these values especially for extended projectile in addition the extended projectile slides within disordered plasma with very low stopping power and without interaction with high target electron .

7.References

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