Description of transitions shape between the dynamic symmetries in $^{152-160}$ Er nuclei

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Abstract:

In the present research the $^{152-160}$ Er Isotopes have been studied by using the interacting boson model -1, From the experimental and calculated energy levels ,electromagnetic transition B(E2),B(E2) ratios(R,R' and R")& $Q2_1^+$ values we consider that there are different modes of behavior to the $^{152-160}$Er isotopes

- SU(5) limit for $^{152-154}$Er.
- SU(5)-O(6) transition region for ^{156}Er.
- O(6)-SU(3) transition region for ^{158}Er.
- SU(3) limit for ^{160} Er.

These chains of isotopes were considered as having transition properties from vibrations to O(6) and a tendency towered more rotational properties with mass of the isotope increase.

وصف شكل الانتقالات بين التماثلات الديناميكية في نوى اليوربيوم (152-160)

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الخلاصة:

في البحث الحالي تمت دراسة نظائر اليوربيوم (152-160) باستعمال نموذج البوزونات المتفاعلة- 1 ومن خلال القيم العملية والنظرية لمستويات الطاقة ,الانتقالات الكهرومغناطيسية (B(E2 ونسب الانتقالات (,R,R' R")وقيم العزم الكهربائي للمستوي 2^+_1 نعتقد أن هناك أنماط مختلفة من السلوك لهذه النظائر

- التحديد الاهتزازي (SU(5) لنظائر $^{152-154}$Er.
- المنطقة الانتقالية بين التحديد (SU(5)-O(6لنظير ^{156}Er .
- المنطقة الانتقالية بين التحديد (O(6)-SU(3لنظير ^{158}E r.
- التحديد الدوراني (SU(3 لنظير ^{160} Er.

اعتبرت هذه السلسلة من النظائر بأنها تمتلك صفات انتقالية من الاهتزازية الى (O(6 وتميل الى الصفات الدورانية اكثر بزيادة العدد الكتلي .

:Introduction

In 2008, J.Sheikh et. al. expand the triaxial projected shell model basis to include triaxially –deformed multi-quasiparticle states .This allows to study the yrast and γ-vibrational band up to high spins for both γ-soft and well –deformed nuclei .It is that γ- band for neutron –deficient isotopes of 156 Er and 158 Er are close to the yrast band ,and further these band are predicted to be nearly degenerate for high –spin states [1].In seam year D.Bonatssos et .al. used the interacting boson model to describe the different phases in several Sm ,Gd, Dy ,Er, U and Fm isotopes[2] .

-:(Interacting Boson Model (IBM

The IBM was created in 1974 by F. Iachello and A. Arima, it has been successfully applied to a wide range of nuclear collective phenomena.

A model of the atomic nucleus has to be able to describe nuclear properties such as spins and energies of the lowest levels, decay probabilities for the emission of gamma quanta's, probabilities (spectroscopic factors) of transfer reactions, multipole moments and so forth.

The interacting boson model (IBM) is suitable for describing intermediate and heavy atomic nuclei. Adjusting a small number of parameters, it reproduces the majority of the low-lying states of such nuclei.

The IBM is based on the well-known shell model and on geometrical collective models of the atomic nucleus. Despite its relatively simple structure, it has proved to be a powerful tool. In addition, it is of considerable theoretical interest since it shows the dynamical symmetries of several nuclei, which are made visible using Lie algebras

The essential idea is that the low energy collective degrees of freedom in nuclei can be described by proton and neutron bosons with spins of 0 and 2. these collective building blocks interact. Different choices of L=0 (s-boson) and L=2 (d-boson) energies and interaction strengths give rise to different types of collective spectra[3-9].

These bosons are interpreted as correlated pairs of protons and correlated pairs of neutrons in the valence shell. This interpretation places restriction on the boson number which is determined by counting the number of particle pairs (separately for protons and neutrons) if the shell is less than half filled. And by counting the number of hole pairs if the shell is more than half filled. If the bosons of neutrons and the bosons of neutrons were considered identical then the interacting boson model is in its simplest form which is called IBM-1[4]

The Hamiltonian of IBM-1 can be given as:

$$
\begin{aligned}
H &= \varepsilon_S n_s + \varepsilon_d n_d + \sum_{L=2,4,6} \frac{1}{2}(2L+1)^{1/2} C_L \left\{ \left[d^\dagger \times d^\dagger\right]^{(L)} \times \left[d \times d\right]^{(L)} \right\}^{(0)} \\
&+ \frac{1}{2^{1/2}} \upsilon_2 \left\{ \left[d^\dagger \times d^\dagger\right]^{(2)} \times \left[d \times s\right]^{(2)} + \left[d^\dagger \times s^\dagger\right]^{(2)} \times \left[d \times d\right]^{(2)} \right\}^{(0)} \\
&+ \frac{1}{2} \upsilon_0 \left\{ \left[d^\dagger \times d^\dagger\right]^{(0)} \times \left[s \times s\right]^{(0)} + \left[s^\dagger \times s^\dagger\right]^{(0)} \times \left[d \times d\right]^{(0)} \right\}^{(0)} \\
&+ u_2 \left\{ \left[d^\dagger \times s^\dagger\right]^{(2)} \times \left[d \times s\right]^{(2)} \right\}^{(0)} \\
&+ \frac{1}{2} u_0 \left\{ \left[s^\dagger \times s^\dagger\right]^{(0)} \times \left[s \times s\right]^{0} \right\}^{(0)} \quad \text{..............................(1)}
\end{aligned}
$$

where n_s and n_d are number operators, ε_s and , ε_d are single boson energies for s- and d boson respectively. The C_L, V_2, V_0, u_2 and u_0 are corresponding interaction parameters.

This form of Hamiltonian is the most direct form which includes all allowed one-body and two-body interactions in the second quantization formalism. Alternatively, another form of Hamiltonian which emphasizes its multipole character is also adopted [9].

$$
H = \varepsilon\ n_d + a_0 P^\dagger P + a_1 L^\dagger L + a_2 Q^\dagger Q + a_3 T_3^\dagger T + a_4 T_4^\dagger\ T_4^\dagger \quad \text{……………....(2)}
$$

Where ε is the boson energy, the parameters a_i's designate the strengths of the, pairing, angular momentum, quadrupole, octupole, and hexadecapole interaction between bosons respectively. A successful nuclear model must yield a good description not only of the energy spectrum of the nucleus but also of its electromagnetic properties.

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The one body transition operator which has the second quantized form is[8] :

$$T_m^{(l)} = \alpha_2 \delta_{l2} \left[d^+ s + s^+ d\right]_m^{(l)} + \beta_l \left[d^+ d\right]_m^{(l)} + \gamma_0 \delta_{l0} \delta_{m0} \left[s^+ s\right]_0^{(0)} \quad \text{.........................(3)}$$

Where α_2,$\beta_l$and γ_0 are coefficient of the various terms in the operator .this equation yields transition operator for E0,M1,E2,M3 and E4 transitions with appropriate values of the corresponding parameters. The most important electromagnetic features are the E2 transitions. The B(E2) values were calculated by using the E2 operator. The E2 transition operator must be a hermitian tensor of rank two and therefore the number of bosons must be conserved. Since, with these constraints the general E2 operator can be written as [7]

$$T_m^{(E2)} = \alpha_2 \left[d^+ s + s^+ d\right]_m^2 + \beta_2 \left[d^+ d\right]_m^2 \quad \text{.........................(4)}$$

The $T_m^{(E2)}$ operator ,which has enjoyed a widespread application in the analysis of γ-Ray transitions .

Calculation :

In the framework of the Interacting Boson Approximation (IBA) model , which describes nuclear structure of even–even nuclei within the U(6) symmetry, possessing the U(5), SU(3), and O(6) limiting dynamical symmetries, appropriate for vibrational, axially deformed, and -unstable nuclei respectively, shape transitions have been studied using the classical limit of the model [4- 7].

Calculation were performed in the complete Hamiltonian using the IBM -1 computer code for energies and IBMT-code for B(E2) values.

For $^{152-160}$Er there are (8-12) active bosons ,the values of the parameters which gave the best fit to experimental data [10-15] are given in tables (1) and fig.(1),for energy levels, and table (2)for B(E2) transition, in figs.(2:a,b,c,d&e) the calculated energy levels are compared with the experimental data[10-15].

.Table(1):The parameters obtained from the programs IBM-1 code

Isotopes	**(Parameters in(Mev**						
	.Eps	P.P	L.L	Q.Q	T3.T3	T4.T4	CHI
Er^{152}	0.971	0.0	0.001	0.0	0.1979-	0.06	0.0
^{154}Er	0.309	0.0	0.001	0.0	0.097	0.06	0.0

^{156}Er	0.987	0.548	0.055	0.0	0.152	0.0	0.0
^{158}Er	0.0	0.343	0.009	0.013-	0.063	0.0	0.01-
Er^{160}	0.0	0.0	0.149	0.013-	0.0	0.0	-1.32

.Table(2):The parameters obtained from the programs IBMT code

Isotopes	Parameters		
	$(B(E2:2_1^+\rightarrow 0_1^+)(e^2b^2$	(E2SD(eb	(E2DD(eb
Er^{152}	0.118	0.1214	0.085-
^{154}Er	0.1896	0.145	-0.1016
^{156}Er	0.338	0.1816	-0.056
^{158}Er	0.559	0.130	0.0
Er^{160}	0.83	0.113	-0.149

The three electromagnetic transition rates. Particularly important are the ratios[9]

$$R=\frac{B(E2:4_1^+\rightarrow 2_1^+)}{B(E2:2_1^+\rightarrow 0_1^+)}$$

$$R'=\frac{B(E2:2_2^+\rightarrow 2_1^+)}{B(E2:2_1^+\rightarrow 0_1^+)}$$

$$R''=\frac{B(E2:0_2^+\rightarrow 2_1^+)}{B(E2:2_1^+\rightarrow 0_1^+)}$$

Which changes from

$R=R'=R''=2[(N-1)/N]$ IN U(5)

to

$R=\frac{10}{7}\frac{(N-1)(2N+5)}{2(2N+3)}\approx 1.4, R'=R''=0$ IN SU(3)

to and

$R=R'=\frac{10}{7}\frac{(N-1)(N+5)}{2(N+4)}\approx 1.4, R''=0$ IN O(6)

are illustration in the fig.(3) ,see table (3) ,while Q_{21}^+ and the experimental[10-15] and calculated $B(E2:2^+_1\rightarrow 0^+_1)$are shown in figs,(4)&(5)

Table(3):Experimental[10-15]B(E2) values (e^2b^2) and $Q2_1^+$ (eb) in $^{152-160}$Er nuclei are compared with IBM-1 results.

i→f	$B(E2)e^2b^2$				
	^{152}Er	^{154}Er	^{156}Er	^{158}Er	^{160}Er

	P.w	Exp.	P.w	Exp.	P.w	Exp.	P.w	Exp.	P.w	Exp.
$2_1^+ \rightarrow 0_1^+$	0.118	0.118	0.189	0.189	0.338	0.338	0.559	0.559	0.83	0.83
$2_1^+ \rightarrow 0_2^+$	0.0	-	0.0674	-	0.0	-	0.0	-	0.0	-
$2_2^+ \rightarrow 0_1^+$	0.0	-	0.0	-	0.158	-	0.0	-	0.0	-
$2_2^+ \rightarrow 0_2^+$	0.0	-	0.008	-	0.0026	-	0.173	-	0.008	-
$2_1^+ \rightarrow 2_2^+$	0.0	-	0.337	-	0.0	-	0.775	-	0.0	-
$4_1^+ \rightarrow 2_1^+$	0.0	-	0.337	-	0.089	-	0.775	-	1.167	-
$4_2^+ \rightarrow 2_1^+$	0.20	-	0.0	-	0.0	-	0.0	-	0.0	-
$4_2^+ \rightarrow 2_2^+$	0.0	-	0.231	-	0.0	-	0.453	-	0.378	-
Q21$^+$	-0.19	-	-0.227	-	-0.078	-	-0.061	-	-2.439	-

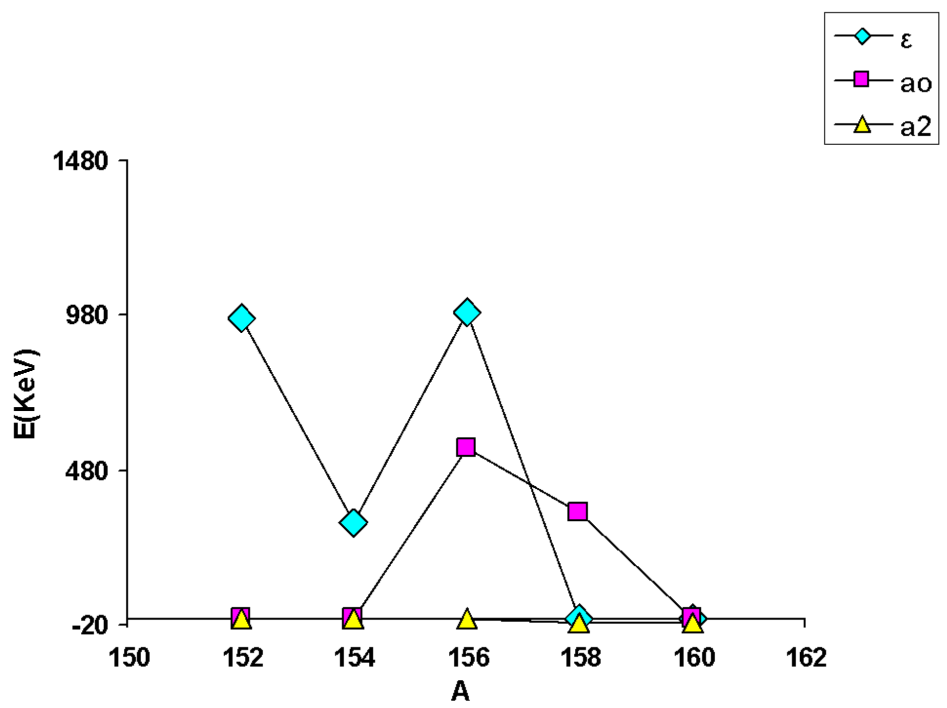


Fig.(2):The values of the (Eps., a0 and a2) parameters which give the best fit to the expermental

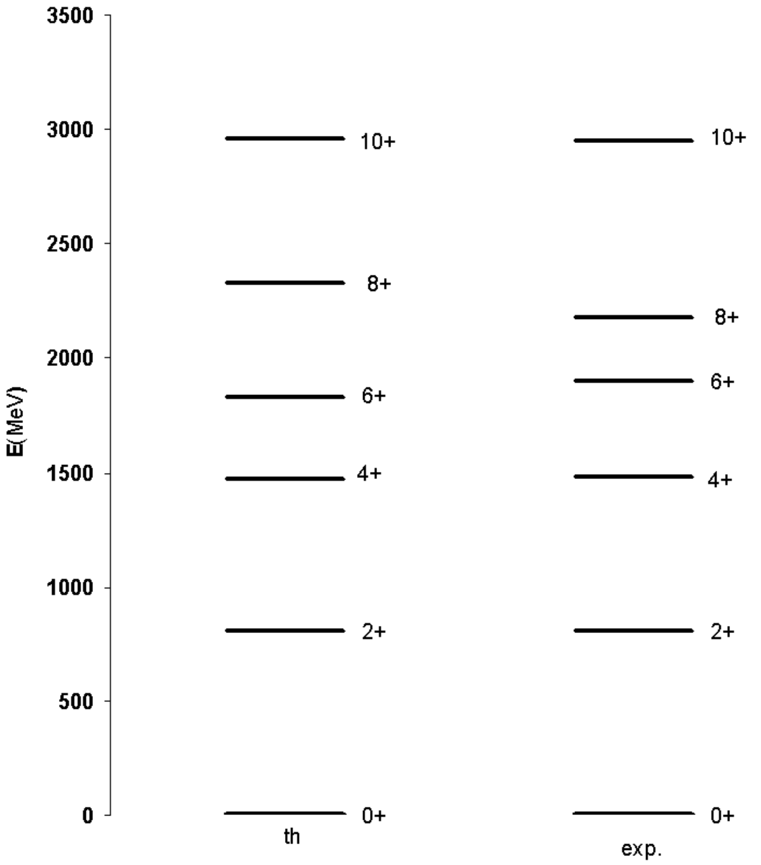


Fig. (2a): Comparsion of experemental [11] and theortical energy levels of 152 Er isotope.

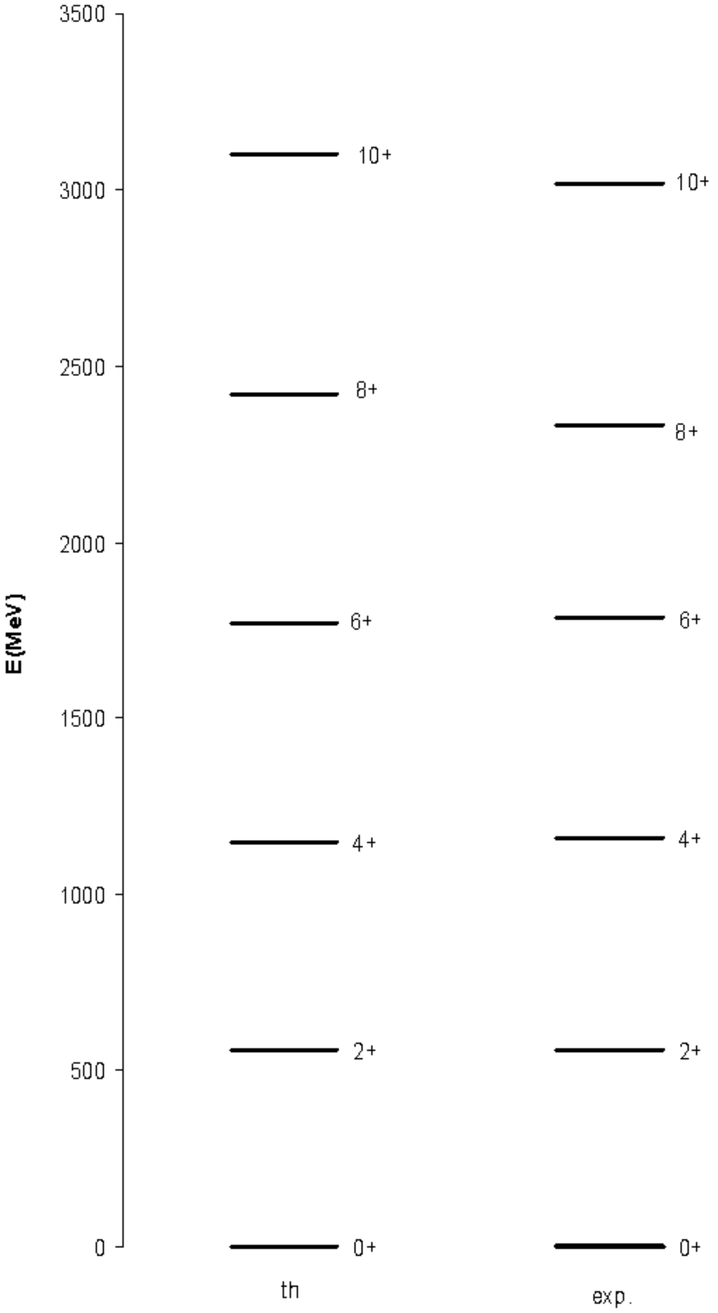


Fig. (2b): Comparsion of experemental [12] and theortical energy levels of 154 Er isotope.

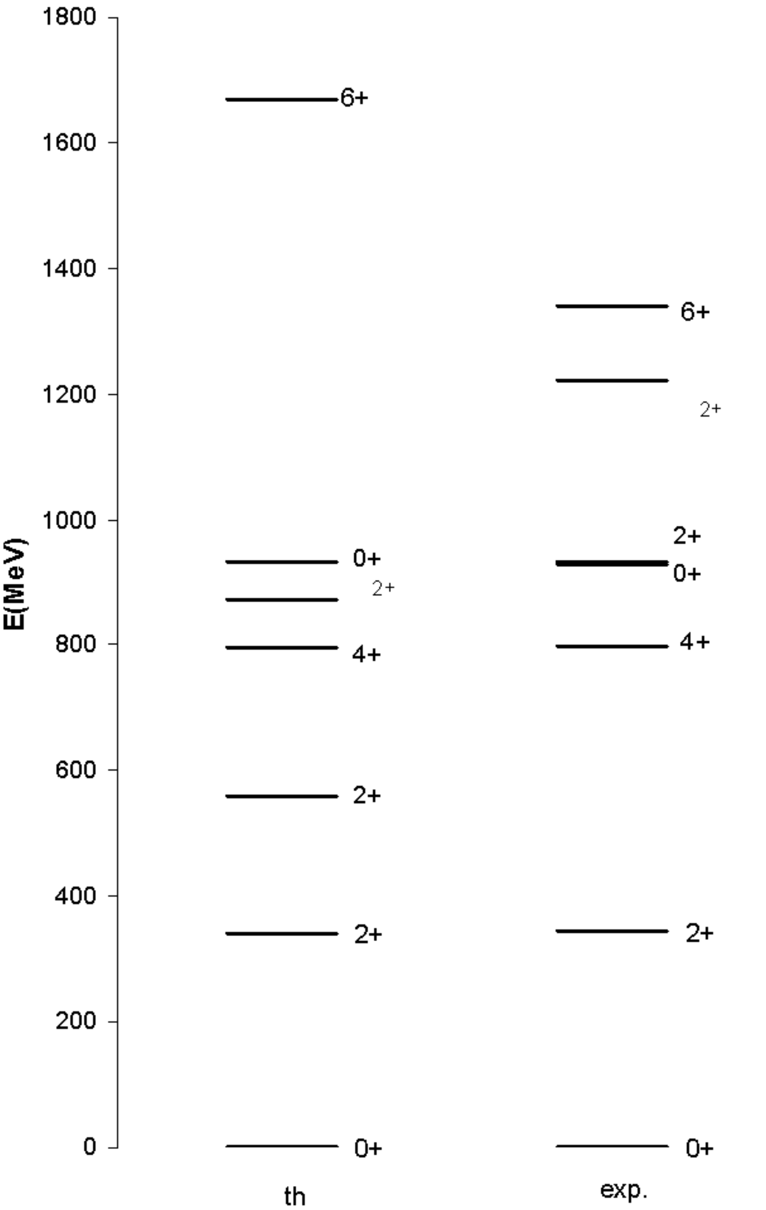


Fig. (2c): Comparsion of experemental [13] and theortical energy levels of 156Er isotope.

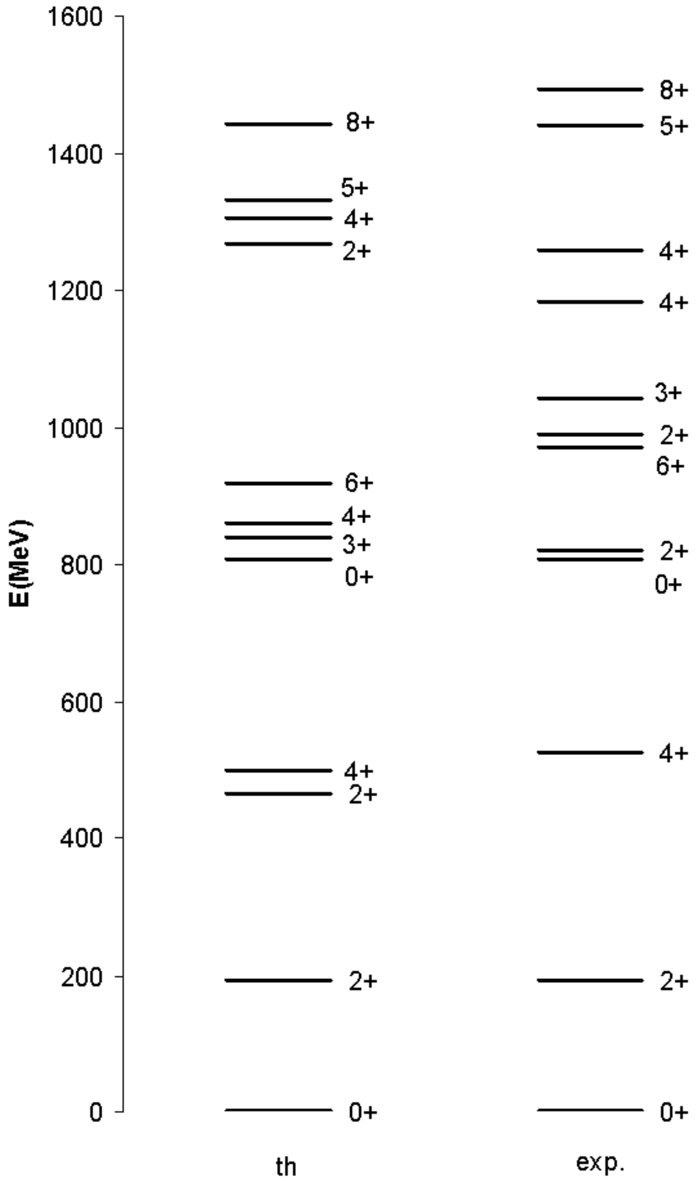
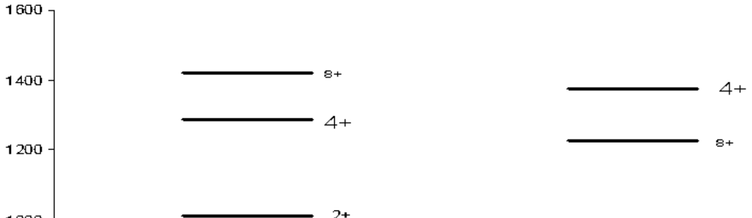


Fig. (2d): Comparsion of experemental [14] and theortical energy levels of 158 Er isotope.



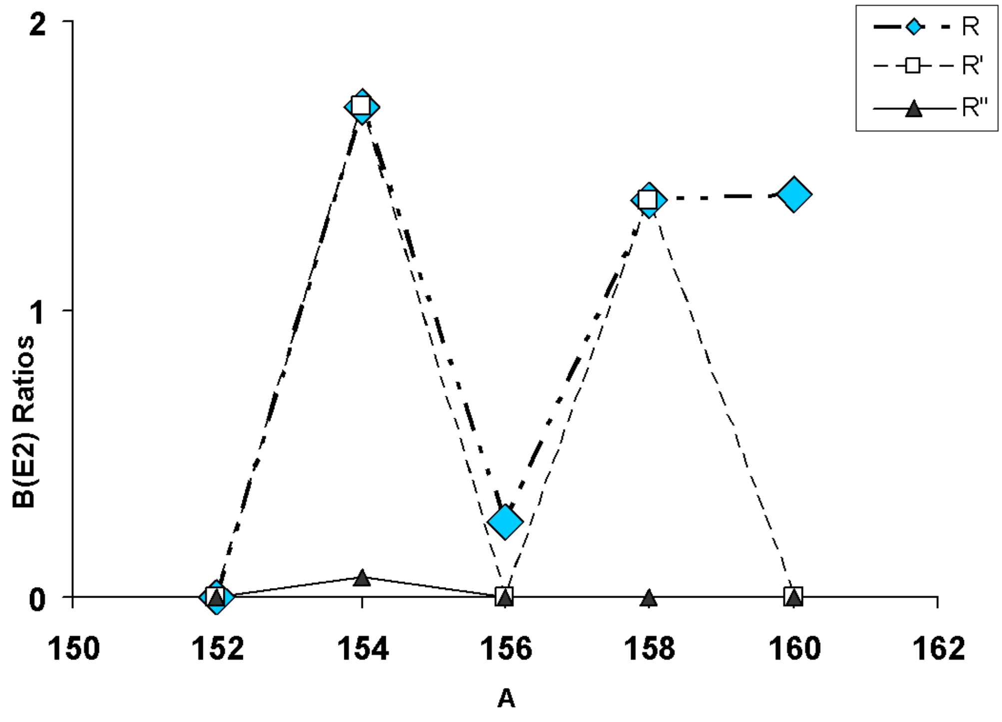


Fig.(3) :Calculated B(E2) Ratios of $^{152-160}$Er isotopes.

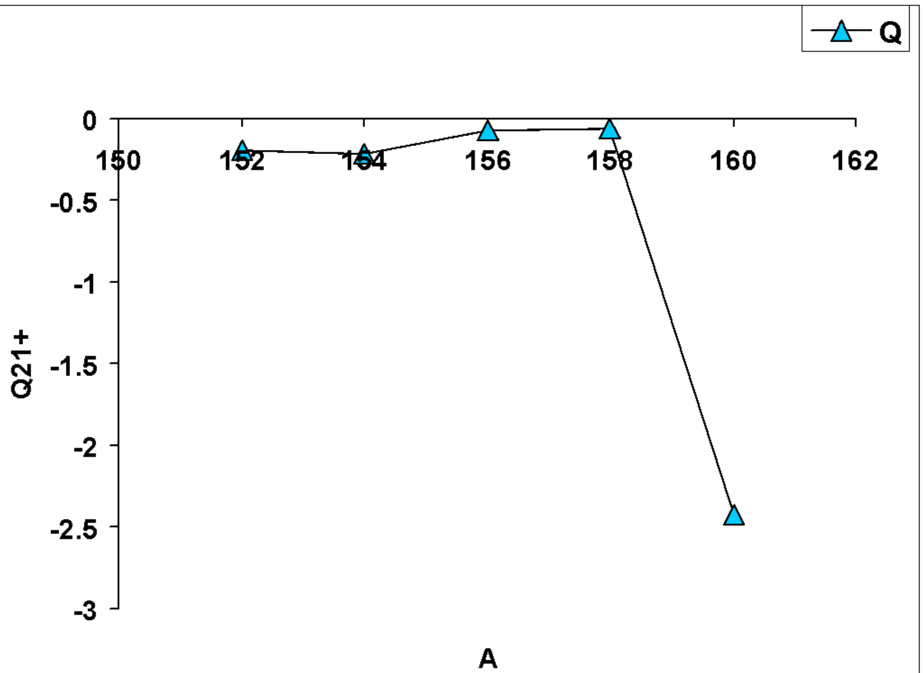


Fig.(4):Calculated electric quadrupole moments in (eb)of the first excited 2+ state.

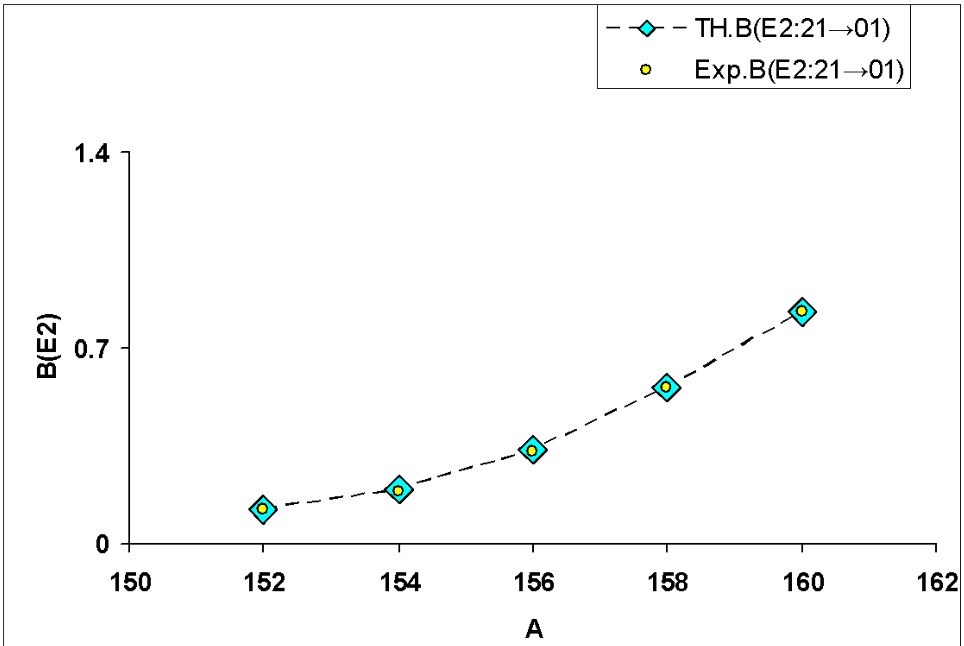


Fig.(5): Calculated and experimental [10-15] Reduced transition probabilities B(E2).

Discussion and conclusion:

The pairing and the quadrupole forces are important in deformed nuclei .These forces especially influence the particles in the unfilled states .The pairing force keeps the nuclei in spherical symmetry .The quadrupole charge distribution causes what is known as the quadrupole force . This force take the nuclei to the deformed state .The relation between the pairing and the quadrupole forces determines the form of the nuclei.

The even-even Erbium isotopes have been described by the IBA-1 Hamiltonian yields a good description of the energy levels in addition to the excitation energies and the electric quadrupole transition probability B(E2; Ii →If) of the $^{152-160}$Er isotopes. The $^{152-160}$Er nuclei have 7 bosons proton(hole) and (1-5)boson neutron (particles) then the total number of bosons is (8-12) respectively.

Since the erbium nucleus has a rathet deformation character ,taken into account the dynamic symmetry location of the even even erbium nuclei at the IBM trangle where their parameter sets are at

- SU(5) for 152-154Er.
- SU(5)-O(6) for 156Er.

- O(6)-SU(3) for 158Er.
- SU(3) for 160 Er.

In the present work and from the first sight we can see that the$^{152-160}$ Er isotopes leave the SU(5) to O(6) toward the SU(3) because of the experimental and calculated ratio values E^{+}_{4}/E^{+}_{2}, E^{+}_{6}/E^{+}_{2} & E^{+}_{8}/E^{+}_{2} which occur near SU(5) for $^{152-154}$ Er and between SU(5)& O(6) for ^{156}Er and between O(6)&SU(3) to ^{158}Er then closer to SU(3) characteristics to^{160}Er isotopes (see figs.(6&7) then, when we look to the B(2) ratios, (fig:3) we can observed the same features .As well as that the quadrupole moments values will be give the same impression where $Q2_1^+$ which measures the deviation of the nuclear charge distribution from a spherical shape advance step by step to O(6) then to SU(3).It's clear that there are different modes of behavior to the $^{152-160}$Er isotopes
as we can see the location on the casten triangle [9] fig.(8).

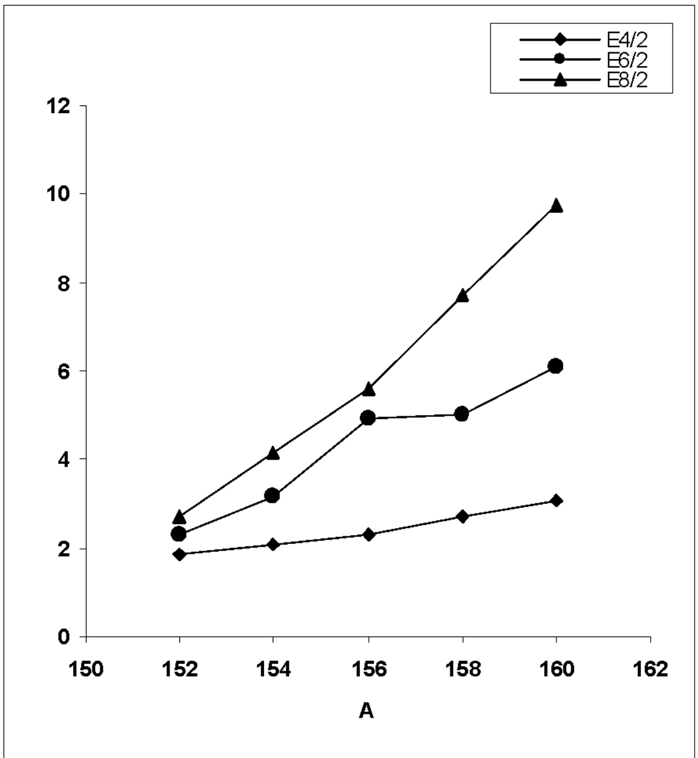


Fig.(6):Experimental [10-15] ratios $R_{4/2}$,R6/2 and $R_{8/2}$ for $^{152-160}$Er isotopes.

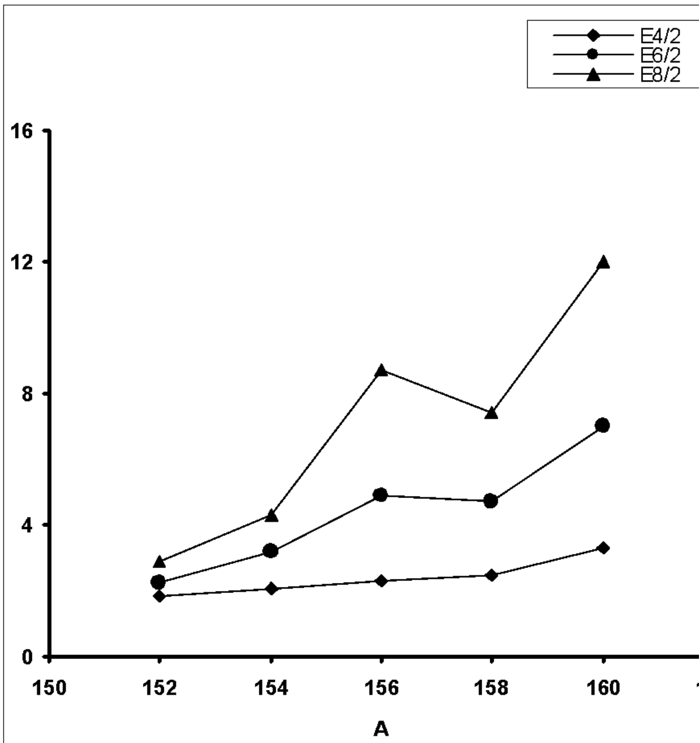
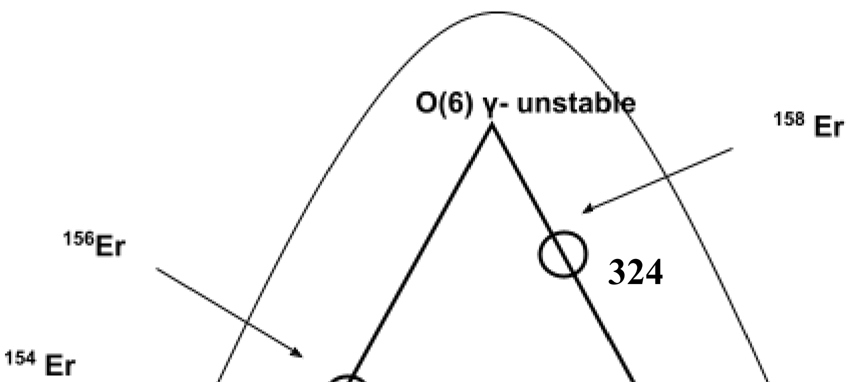


Fig.(7):Calculated ratios R4/2,R6/2 and R8/2 for152-160Er isotopes.



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