Alxandroff Regular Generalization Sets

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Abstract : In this paper introduced the concept of ***Alxandroff Regular Generalization sets*** (briefly A-g-regular sets) and investigate some of its properties, Since introduce A-g-regular closed sets and investigate some of its consequences and A-g-regular .open sets

الخلاصة :- في هذا البحث قدمنا مفهوم المجموعات العمومية في فضاء الكساندروف المنتظم وناقشنا بعض خواصه حيث قدمنا مفهوم المجموعات المغلقة العامة والمجموعات المفتوحة العامة وناقشنا بعض النتائج والمبرهنات حولهما.

Introduction-1

In 1999 (F.G. Arenas)[2], introduced Alxandroff space (a topological space such that every point has minimal neighborhood). Recently, Pratvlanapa [4] has introduced the concept of generalized closed sets of Alxandroff space. In this paper, we obtain new concept of A-g-regular sets We investigate various properties of this concept and .obtain A-g-regular sets in this space

Definition 1-1 [1][2] : An Alxandroff space (briefly A-space) is a set X together : with system τ of subsets satisfying the following

i) X and ϕ are in τ.

ii) The intersection of a finite number of a sets τ is a set in τ.

iii) the union of any countable number of sets from τ is a set in τ.

the pair (X, τ) is called A-space, the members of τ are called A-open set, and their .complementary sets are called A-closed

Definition1- 2 [3]: A subset M of topological (X, T) is called generalized closed set (briefly g-closed set) if $cl(M)\subseteq U$ whenever $M\subseteq U$ and U is open set, the . (complement of M is called generalized open set (briefly g-open set

Definition1- 3 **:** (X,τ) is called A-regular set if for any $x\in X$ and any A-closed set F such that $x\notin F$ there exist A-open sets $U,V\in\tau$ such that $x\in U, F\subseteq V$ and $U\cap V=\phi$.

Definition1- 4 :- A subset M of A-space *X* is A- compact set if every open cover of M is reducible to finite cover

2- A- Regular Generalization Closed Sets

***Definition* 2-1:** A subset *M* of *X* is called a A-Regular Generalization closed set (briefly A-g-regular closed set) if there is A-closed set *F* contain *M* such that $F\subset U$ .whenever $M\subset U$, where *U* is A-open and A-regular set

Theorem2-2 **:** If M is A- g- closed set then M is A-g- Regular closed set.

Proof : Suppose that $M\subset U$, when U is A - Regular open set,

,since M is A- g- closed set

,then there is a A-closed set F containing M

, $F \subset U$ such that

• and this shows that M is A-g- Regular closed set

<u>Remark 2-3</u> : Every A- g- closed set is A-g- Regular closed set but the converse of is .not true as shown by the following example

<u>Example 2-4 :-</u> let X={a, b, c, d} with τ ={ Φ ,{a}, {b}, {a, b}, {a, b, c}, {a, b, d}, {X} and let M={a, c

, $M \subset \{a,b,c\}$ its clear that

since {a, b, c} is A-open

, $F \not\subset \{a,b,c\}$ and F={a, c, d} is A-closed contain M and

,this implies M is not A- g- closed set

,since X is the only A- Regular open contain M

, $F \subset X$ since $M \subset X$ and

.therefore M is A-g- Regular closed

<u>Theorem 2-5</u> : If M and N are A-g- Regular closed sets then $M \cup N$ is A-g- Regular closed set

,***<u>Proof:</u>*** Let $(M \cup N) \subset U$ and U BE A- Regular open set

. $N \subset U$ then $M \subset U$ and

,since M and N are A-g- Regular closed sets

and hence there is F and H are A-closed sets contain M and N respectively, such that

, $H \subset U$ $F \subset U$ and

, $F \cup H \subset U$ hence

,since F and H are A-closed sets

$M \cup N$ then $F \cup H$ is A-closed contain

• and hence $M \cup N$ is A-g- Regular closed set

<u>Theorem 2-6 :</u> Let $N \subset M \subset X$, N be A-g- Regular closed set relative to M where M is A-g- Regular closed set and A-open subset of X , then N is A-g- Regular closed set .relative to X

,***<u>Proof:</u>*** Let $N \subset U$ where U is A- Regular open set

$N \subset M \cap U$ we have

,Such that $M \cap U$ is A- Regular open set in M

,since N is A-g- Regular closed set relative to M

,hence there is F is A-closed in M contain N

,such that $F \subset M \cap U$, since F is close in M

,then there is H is closed in X contain N

. $F = H \cap M$ such that

, $F = H \cap M \subset M \cap U$ This means

$F = H \cap M \subset U$, this implies that

. $M \cap (H \cup H^C) \subset U \cup H^C$ and hence

, $H \cup H^C = X$ Since

, $(M \cap X) \subset (U \cup H^C)$ we have

$M \subset (U \cup H^C)$. so

,Since H^C and U are A-open sets

,then $(U \cup H^C)$ is A-open set

,since M is A-g-closed in X

,then there is closed set G contain M

$G \subset (U \cup H^C)$. such that

$H \subset (U \cup H^C)$ But $H \subset G$, then

$H \subset G$, and hence

- we have N is A-g- Regular closed set relative to X

Corollary 2-7 :- If M is a A-g- closed and open set and F is a A-closed set in X then $F \cap M$. is a A-g- Regular closed set in X

,***Proof:*** since F is A-closed in X then $F \cap M$ is A-closed in M

,let $F \cap M \subset U$ where U is Regular open in M

$F \cap M$, Then there is A-g-closed set H contain

$H \subset F \cap M \subset U$. Such that $H \subset U$ and so

,Hence $F \cap M$ is A-g- Regular closed set in the A-g- closed set M

(therefore by theorem (2-6

- we have $F \cap M$ is A-g- Regular closed set in X

Theorem 2-8 :- If a set M is A-g- Regular closed set then there is A-close set contain .M such that F-M contain no nonempty Regular closed set

Proof :- Let M be A-g- Regular closed set and F be closed set contain M

$H \subset F - M$, Such that

where H is A-g- Regular closed set

$H \subset F \cap M^C$, then

$M \subset H^C$, so

,but M is A-g- Regular closed set

$F \subset H^C$, therefore

,since H^C is A-g- Regular open set

$H \subset F$ consequently $H \subset F^C$ since we have

$H \subset F^C \cap F = \phi$ and hence

- Thus H= ϕ , therefore F-H contain no nonempty Regular closed set

.***Remark 2-9*** :- The converse of the theorem is not true

For example : in example (2-4) there is close set contain {a} say {a, c, d } but {a, c, d}-{a}={c, d} dose not contain non-empty A- Regular closed set , and {a} is not A-g- , Regular closed set in X

$\{a\} \subset \{a\}$, since

$\{a\} \subset \{a,c,d\} \not\subset \{a\}$. but there is F is closed say

Corollary 2-10 :- Let M be a A-g- Regular closed set, then M is regular and closed set if and only if there is U is A-open contain M and F-M is regular and A-closed where F is A-closed contain U

,***Proof*** :- Suppose that M is A-g- Regular closed set
if M is regular and closed set
,ϕ =since we have M=F, then F-M
but ϕ is always regular and closed set
,therefore F-M is regular and closed set
,Conversely, suppose that F-M is regular and closed
,Since M is A-g- Regular closed set
,Then H-M contain F-M where H is closed contain M
,ϕ =Since F-M is closed we have F-M

- Hence F=M, therefore M is regular and closed set

Theorem 2-11 **:-** If M is A-g- Regular closed set and $M \subset N \subset \overline{M}$ then $\overline{N} - N$ contains no nonempty regular closed set

Proof:- Since $M \subset N$ then $N^c \subset M^c$ ... (1
,and M is A-g- Regular closed set,
,$F \subset U$ Then there is F is closed set contain M such that
,$N \subset \overline{M} \subset F$ where U is A-regular open set : now
and hence $\overline{N} \subset \overline{F}$ =F ….. (2
$\overline{N} \subset F$ since F is A- closed
,$\overline{N} \cap N^C \subset F \cap M^C$ from 1 and 2 we have
,$\overline{N} - N \subset F - M$ and
,since M is A-g-closed set
then F-M contain no nonempty regular closed set

- and hence $\overline{N} - N \subset F - M$ contain no nonempty regular closed set

.***Theorem 2-12*** :- Let $M \subset Y \subset X$ where M be A-g- Regular closed set in X Then M is A-g- Regular closed set relative to Y, where Y is .A-open in X

.***Proof*** :- Let $M \subset Y \cap U$ where U be regular open in X
.Then $M \subset U$ and hence $F \subset U$ where F is closed set contain M
.$Y \cap F \subset Y \cap U$ Then

- Thus means M is A-g- Regular closed set relative to Y

Lemma 2-13 [3] **:-** If A is compact regular space in X , such that $A \subset U$, Then there $A \subset V \subset \overline{V} \subset U$ is open set V such that

Theorem 2-14 :- let X be A-Regular space in X and let M be an A-compact subset of .X. Then M is A-g- Regular closed set

,***Proof:-*** Suppose that $M \subset U$ where U is A-regular open set in X
.since U is A-open , But M is A-compact in the A-regular space X
(Then there is A-open set V such that $M \subset V \subset \overline{V} \subset U$ by (lemma 2-13
.And hence $H \subset \overline{\overline{V}} = \overline{V}$ where H is A-closed contain M

$H \subset \overline{V} \subset U$ We get

. $H \subset U$ and hence

- Then M is A-g- Regular closed set

A- Regular Generalization open sets -3

Definition 3-1 **:-** A subset *M* in an A-space *X* will be called A-Regular Generalization .open set (briefly A-g-regular open set) if $X - M = M^C$ is A-g-regular closed set

Theorem 3-2 :- A set M is A-g-regular open set if and only if the following condition $F \subset M$ hold: $F \subset U$ where U is A-open in M and F is regular closed and

Proof:- Suppose that : $F \subset U$ where U is A-open in M and F is A-regular closed and $F \subset M$

Let $M^C = N$ such that $N \subset U$ were U be A-regular open set

Now $M^C \subset U$ so $F = U^C \subset M$ where F is A-regular closed set

.Which implies $F \subset V$ where V is A-open contain in M

, $V^C \subset F^C = U$ We have

and hence $H^C \subset U$ where H is open contain in M

,So H^C is A-closed contains N

,such that U is A-g-regular closed contain N

.and hence N is A-g-regular closed set

,We have M is A-g-regular open set

.Suppose that M is A-g-regular open set

, Let $F \subset M$ where F be A-regular closed set

,then F^C is A-regular open set

. $M^C \subset F^C$ and

, M^C Hence $H \subset F^C$,where H is A-closed contain

- we have $F \subset H^C = U$ where U is A-open contain M

Remark 3-3 **:-** The union of two A-g-regular open sets is not necessarily to be ,A-g-regular open set

Since: Example (2-4) show that {b, c} and {d} are A-g-regular open sets

.But their union say {b, c, d} is not A-g-regular open set

$N \cup M$ ***Theorem 3-4*** :- If N and M are separated A-g-regular open set then so

, $N \cup M$ ***Proof*** :- Let F be A –regular closed subset of

,then $F \subset M$ and $F \cap H \subset M$ where H is closed subset of M

,and hence $F \cap H \subset U$ where U is open subset of M

similarly $F \cap G \subset V$ where G is closed subset of N and V is open subset of N, then

. $\subset U \cup V$ $F = F \cap (N \cup M) \subset (F \cap H) \cup (F \cap G)$

. $N \cup M$ Where $U \cup V$ is open contain

- Hence $F \subset U \square V$ and $N \square M$ is separated A-g-regular open set

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