On Pairwise Fibers in Bitopological Spaces

By
Khalid Shea.Al-Shukree
Department of mathematics, College of Education,Al-Qadisiyh University

-:Abstract

In this work, we give the certain types of mapping with certain types of pairwise fibers and study the several properties of each type .Also; we study the relation between these mappings. These mappings are novel in the present time at the best of our knowledge. Also, we give the definitions of certain types of pairwise light open .property and investigate its several properties

-:I. Introduction

The study of bitopological spaces was introduced by Kelly in [4] and thereafter a large number of papers have been done to generalize the topological concepts to bitopological spaces. Khalid in [7] introduced the concept of δ-light mapping the mapping with totally δ-disconnected fibers. In this work ((X,τ_1, τ_2)and (Y,σ_1,σ_2) or briefly X,Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological spaces X, by A^{o_i} and $\overline{A}^{i}$ we denoted respectively the interior of A and the closure of A with respect to τ_i(or σ_j) for i=1,2. Also i,j=1,2 for i≠j. The mapping f mean a mapping from a bitopological space X onto a bitopological Y , $f^{-1}(y)$ is called the pairwise fibers for .all y ∈ Y

-:II. Preliminaries

.In this section we introduced some definitions which we needed in our work

II.1Definition:- A subset A of a bitopological space X is called

a) regular –open (R-open)[1] if $A = (\overline{A}^{2})^{o_1}$

b) semi-open (s-open)[8] if $A \subset \overline{(A^{o_1})}^{2}$

c) pre-open[3] if $A \subset (\overline{A}^{2})^{o_1}$

d) α-open[6] if $A \subset (\overline{(A^{o_1})}^{2})^{o_1}$

.e) semi-regular (sR-open)[2] if A is both semi-open and semi-closed

II.2 Definitions:- A mapping $f: X \to Y$ is said to be

.α-continuous if each α-open subset B of Y , $f^{-1}(B)$ is open set in X (1

.S-continuous[8] if each S-open subset B of Y, $f^{-1}(B)$ is open set in X (2

.R-continuous[3] if $f^{-1}(B)$ is regular open in X for every regular open set B of Y (3

.pre-continuous[8] if each pre-open set B of Y , $f^{-1}(B)$ is open set in X (4

α-open resp.(α-closed) if each open set U resp.(closed) in X , *f(*U) is α-open resp.((5 . α-closed)set in Y

S-open resp.(S-closed) if each open set U resp. (closed set) *f(U)* is S-open (6
.resp.S-closed set in Y
pre-open (resp. pre-closed)[5] if each open set U in X (resp.closed set in X) , f(U) (7
.is pre-open (resp.pre-closed) set in Y
R-open (resp. R-closed)[3] if each R-open set U in X (resp.R-closed set in X) , (8
.f(U) is R-open (resp.R-closed) set in Y

-:Then from the definitions above we give the following definitions

<u>II.3 Definitions:-</u> A mapping $f: X \rightarrow Y$ is said to be
α-homeomorphism if (1
.a) f is bijective mapping
.b) f is continuous mapping
.c) f is α- open or α-closed mapping
S- homeomorphism if (2
.a) f is bijective mapping
.b) f is continuous mapping
.c) f is S- open or S-closed mapping

pre - homeomorphism if (3
.a) f is bijective mapping
.b) f is continuous mapping
.c) f is pre- open or pre-closed mapping

R - homeomorphism if (3
.a) f is bijective mapping
.b) f is continuous mapping
.c) f is R- open or R-closed mapping

<u>-:III. Pairwise Disconnection Set in Bitopological Spaces</u>

In this section we give new concepts such as:- α-pairwise disconnection , pre-pairwise disconnection, R-pairwise disconnection, s-pairwise disconnection, sR-pairwise disconnection, and s- α-pairwise disconnection.Also, we study the .relation between these concepts

<u>III.1 Definition:-</u> Let X be a bitopological space and A,B be non-empty disjoint α-open subset of X such that A is τ_1- α-open and B is τ_2- α-open then we say that
. $A \cap B = \phi$ A/B is α-pairwise disconnection to X if $X = A \cup B$ and (1
X is α-pairwise disconnected space if there exists a α-pairwise disconnection A/B (2
. to X , otherwise we say that X is α-pairwise connected space

<u>III.2 Definition:-</u> Let X be a bitopological space and A,B be non-empty disjoint s-open subset of X such that A is τ_1- s-open and B is τ_2- s-open then we say that
. $A \cap B = \phi$ A/B is s-pairwise disconnection to X if $X = A \cup B$ and (1
X is s-pairwise disconnected space if there exists a s-pairwise disconnection A/B to (2
. X , otherwise we say that X is s-pairwise connected space

III.3 Definition:- Let X be a bitopological space and A,B be non-empty disjoint pre-open subset of X such that A is τ_1- pre-open and B is τ_2- pre-open then we say that

. $A \cap B = \phi$ A/B is pre-pairwise disconnection to X if $X = A \cup B$ and (1

X is pre-pairwise disconnected space if there exists a pre-pairwise disconnection (2

. A/B to X , otherwise we say that X is pre-pairwise connected space

III.4 Definition:- Let X be a bitopological space and A,B be non-empty disjoint R-open subset of X such that A is τ_1- R-open and B is τ_2- R-open then we say that

. $A \cap B = \phi$ A/B is R-pairwise disconnection to X if $X = A \cup B$ and (1

X is R-pairwise disconnected space if there exists a R-pairwise disconnection A/B (2

. to X , otherwise we say that X is R-pairwise connected space

III.5 Definition:- Let X be a bitopological space and A,B be non-empty disjoint sR-open subset of X such that A is τ_1- sR-open and B is τ_2- sR-open then we say that

. $A \cap B = \phi$ A/B is sR-pairwise disconnection to X if $X = A \cup B$ and (1

X is sR-pairwise disconnected space if there exists a sR-pairwise disconnection (2

. A/B to X , otherwise we say that X is sR-pairwise connected space

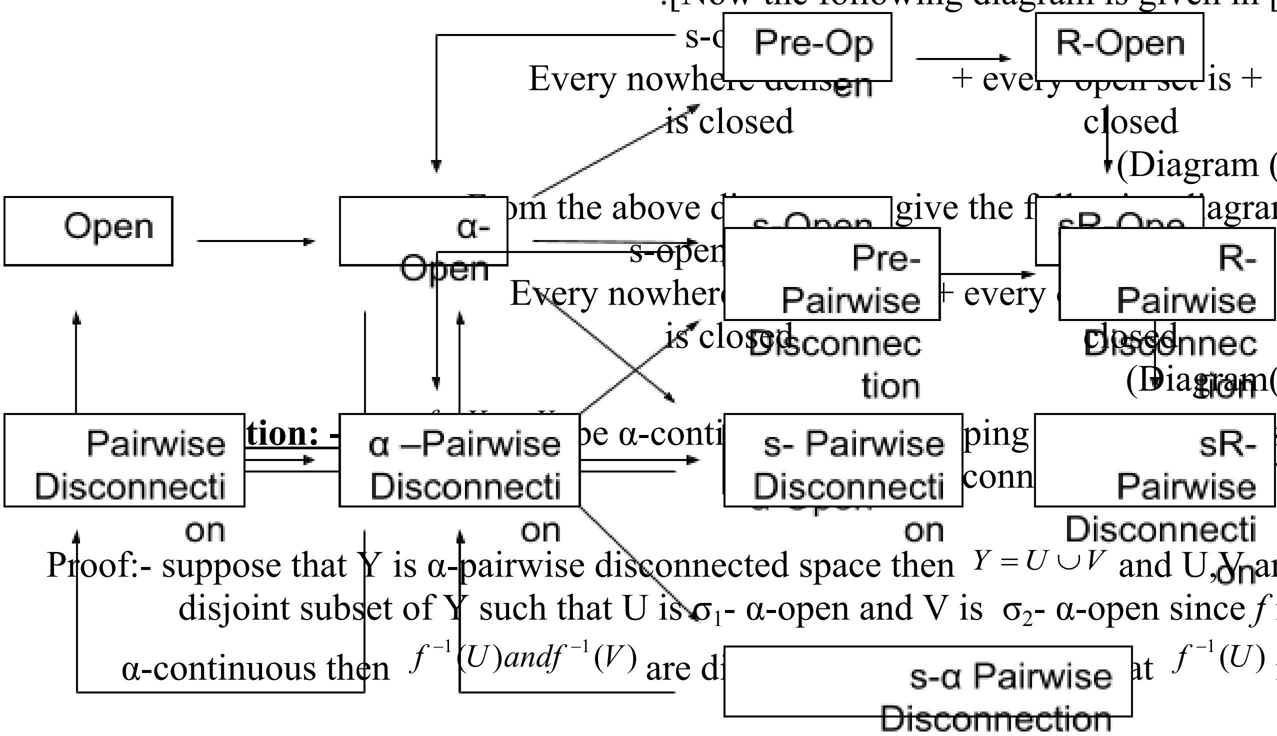
III.6 Definition:- Let X be a space and G be a non-empty subset of X and A,B be non-empty α-open sets in X with respect to τ_1 and τ_2 (rsep.s-open , pre-open , R-open, sR-open)sets in X , then A/B is said to be α-pairwise disconnection (resp. s-pairwise disconnection , pre-pairwise disconnection , R- pairwise disconnection , sR- pairwise disconnection) of G if

$(A \cap G) \cup (B \cap G) = G.$ (1

$(A \cap G) \cap (B \cap G) = \phi.$ (2

Otherwise G is α-pairwise connected set(resp.s-pairwise connected, pre- pairwise .(connected, R- pairwise connected,sR- pairwise connected

.[Now the following diagram is given in [9



s-o... Every nowhere dense ... + every open set is + is closed closed

(Diagram (1

From the above d... give the f... diagram

s-open Every nowhere ... + every ... is closed closed

(Diagram(2

...tion: - ... be α-conti... ping ... se ... conn ... Y

Proof:- suppose that Y is α-pairwise disconnected space then $Y = U \cup V$ and U,V are disjoint subset of Y such that U is σ_1- α-open and V is σ_2- α-open since *f* is α-continuous then $f^{-1}(U) and f^{-1}(V)$ are di... at $f^{-1}(U)$ is

τ_1-α-open and $f^{-1}(V)$ is τ_2- α-open and $f^{-1}(U) \cup f^{-1}(V) = f^{-1}(U \cup V) = f^{-1}(Y) = X$ and this .contradiction with X is α-pairwise connected □

III.8 proposition: - Let $f : X \to Y$ be s-continuous onto mapping if X is s-pairwise .connected then so is Y

.((Proof:- It is easy from definitions (III.2 and II.2 (2 □

III.9 proposition: - Let $f : X \to Y$ be R-continuous onto mapping if X is R-pairwise .connected then so is Y

.((Proof:- It is easy from definitions (III.4 and II.2 (3 □

III.10 proposition: - Let $f : X \to Y$ be pre-continuous onto mapping if X is pre .-pairwise connected then so is Y

.((Proof:- It is easy from definitions (III.3 and II.2 (4 □

III.11 proposition:- Every pre-pairwise connected space is α-pairwise connected .space

Proof:- Suppose that X is α-pairwise disconnected space then there exists a α-pairwise disconnection A/B to X , such that A is τ_1-α-open and B is τ_2-α-open and since every α-open is pre-open then there exists a pre-paiwise disconnection to X and this a .contradiction with X is pre-pairwise connected then X is α-pairwise connected space

.**III.12 proposition:-** Every s-pairwise connected space is α-pairwise connected space

Proof:- Suppose that X is α-pairwise disconnected space then there exists a α-pairwise disconnection A/B to X , such that A is τ_1-α-open and B is τ_2-α-open and since every α-open is s-open then there exists a s-paiwise disconnection A/B to X and this a .contradiction with X is s-pairwise connected then X is s-pairwise connected space □

III.13 proposition:- Every R-pairwise connected space is pre-pairwise connected .space

Proof:- Suppose that X is pre-pairwise disconnected space then there exists a pre-pairwise disconnection A/B to X , such that A is τ_1-pre-open and B is τ_2-pre-open and since every pre-open is R-open then there exists a R-paiwise disconnection A/B to X and this a contradiction with X is R-pairwise connected then X is pre-pairwise .connected space □

III.14 proposition:- Every sR-pairwise connected space is R-pairwise connected .space

Proof:- Suppose that X is R-pairwise disconnected space then there exists a R-pairwise disconnection A/B to X , such that A is τ_1-R-open and B is τ_2-R-open and since every R-open is sR-open then there exists a sR-paiwise disconnection A/B to X and this a contradiction with X is sR-pairwise connected then X is R-pairwise .connected space □

-:IV. Certain Types of Totally Pairwise Disconnected Fibers

In this section we give new concepts such as:- α-pairwise light mapping , pre-pairwise light mapping, R-pairwise light mapping, s-pairwise light mapping, sR-pairwise light mapping, and s- α-pairwise light mapping. Also, we study the .relation between these concepts

IV.1Definition:- Let X be a bitopological space then X is called totally α-pairwise disconnected space if for each pair of points a,b ∈ X there exists a α-pairwise . disconnection A/B to X such that a ∈ A and b ∈ B

IV.2Definition:- Let X be a bitopological space then X is called totally pre-pairwise disconnected space if for each pair of points a,b ∈ X there exists a pre-pairwise . disconnection A/B to X such that a ∈ A and b ∈ B

IV.3Definition:- Let X be a bitopological space then X is called totally s-pairwise disconnected space if for each pair of points a,b ∈ X there exists a s-pairwise . disconnection A/B to X such that a ∈ A and b ∈ B

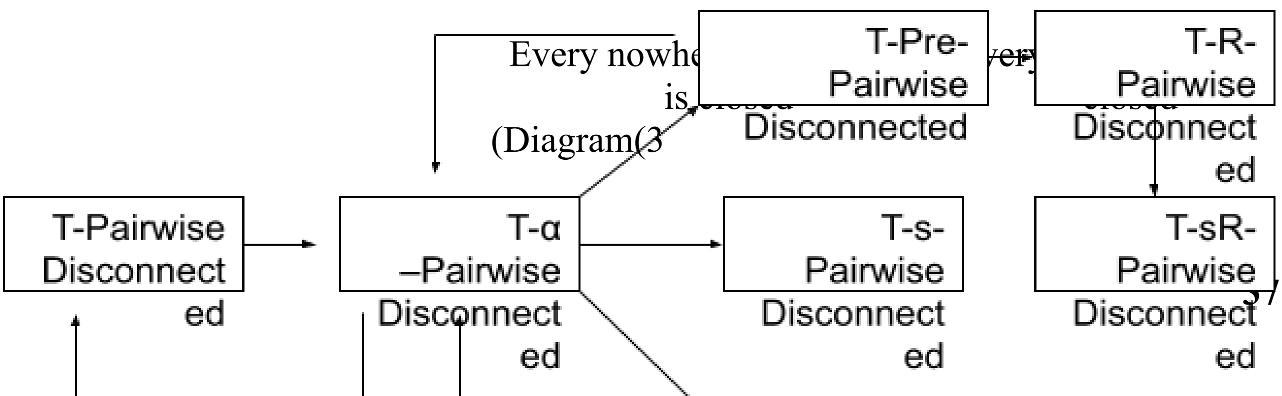
IV.4Definition:- Let X be a bitopological space then X is called totally R-pairwise disconnected space if for each pair of points a,b ∈ X there exists a R-pairwise . disconnection A/B to X such that a ∈ A and b ∈ B

IV.5Definition:- Let X be a bitopological space then X is called totally sR-pairwise disconnected space if for each pair of points a,b ∈ X there exists a sR-pairwise . disconnection A/B to X such that a ∈ A and b ∈ B

IV.6Definition:- Let X be a bitopological space then X is called totally s-α-pairwise disconnected space if for each pair of points a,b ∈ X there exists a s-α-pairwise . disconnection A/B to X such that a ∈ A and b ∈ B

IV.7 Remark:- By the diagram(1) and diagram(2) now we give the relation between -:the definition above from the following diagram

s-open+



As a consequence of definitions [II.2 ,(1),(2),(3),(4)]we have that each of (totally α-pairwise disconnected , totally s-pairwise disconnected , totally pre -pairwise disconnected and totally R-pairwise disconnected) are resp. a topological property.

IV.8Proposition: Let X and Y be two spaces and

a- let $f:X\rightarrow Y$ be a α-homeomorphism .If X or Y is a totally α-pairwise disconnected so is the other.

b- let $f:X\rightarrow Y$ be a s-homeomorphism .If X or Y is a totally s-pairwise disconnected so is the other.

c- let $f:X\rightarrow Y$ be a pre -homeomorphism .If X or Y is a totally pre-pairwise disconnected so is the other.

d- let $f:X\rightarrow Y$ be a α-homeomorphism .If X or Y is a totally α-pairwise disconnected so is the other.

Proof: (a)- Suppose that X is totally α-pairwise disconnected and let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$ .since f is bijective mapping then there exists two points $x_1, x_2 \in X$ such that $x_1 \neq x_2$ and $f(x_1)=y_1$, and $f(x_2)=y_2$ since X is totally α-pairwise disconnected space , then there is a α-pairwise disconnection U/V such that $x_1 \in U, x_2 \in V$ since f is α-homeomorphism then each of $f(U)$ and $f(V)$ are α-open sets in Y but $f(U) \cup f(V) = f(U \cup V) = f(X) = Y$ and since f is one-to-one then $f(U) \cap f(V) = f(U \cap V) = f(\phi) = \phi, \in X$ and $y_1 \in f(U), y_2 \in f(V)$ and hence Y is totally α-pairwise disconnected space. □

b)- Suppose that X is totally s-pairwise disconnected and let $y_1, y_2 \in Y$ such that ($y_1 \neq y_2$ .since f is bijective mapping then there exists two points $x_1, x_2 \in X$ such that $x_1 \neq x_2$ and $f(x_1)=y_1$, and $f(x_2)=y_2$ since X is totally s-pairwise disconnected space , then there is a s-pairwise disconnection U/V such that $x_1 \in U, x_2 \in V$ since f is s-homeomorphism then each of $f(U)$ and $f(V)$ are s-open sets in Y but $f(U) \cup f(V) = f(U \cup V) = f(X) = Y$ and since f is one-to-one then $f(U) \cap f(V) = f(U \cap V) = f(\phi) = \phi, \in X$ and $y_1 \in f(U), y_2 \in f(V)$ and hence Y is totally s-pairwise disconnected space. □

c)- Suppose that X is totally pre-pairwise disconnected and let $y_1, y_2 \in Y$ such that ($y_1 \neq y_2$ .since f is bijective mapping then there exists two points $x_1, x_2 \in X$ such that $x_1 \neq x_2$ and $f(x_1)=y_1$, and $f(x_2)=y_2$ since X is totally pre-pairwise disconnected space , then there is a pre-pairwise disconnection U/V such that $x_1 \in U, x_2 \in V$ since f is pre-homeomorphism then each of $f(U)$ and $f(V)$ are pre-open sets in Y but $f(U) \cup f(V) = f(U \cup V) = f(X) = Y$ and since f is one-to-one then $f(U) \cap f(V) = f(U \cap V) = f(\phi) = \phi, \in X$ and $y_1 \in f(U), y_2 \in f(V)$ and hence Y is totally pre-pairwise disconnected space. □

d)- Suppose that X is totally R-pairwise disconnected and let $y_1, y_2 \in Y$ such that ($y_1 \neq y_2$ .since f is bijective mapping then there exists two points $x_1, x_2 \in X$ such that $x_1 \neq x_2$ and $f(x_1)=y_1$, and $f(x_2)=y_2$ since X is totally R-pairwise disconnected space , then there is a R-pairwise disconnection *U/V* such that $x_1 \in U, x_2 \in V$ since *f* is R-homeomorphism then each of *f(U)* and *f(V)* are R-open sets in *Y* but $f(U) \cup f(V) = f(U \cup V) = f(X) = Y$ and since *f* is one-to-one then $f(U) \cap f(V) = f(U \cap V) = f(\phi) = \phi, \in X$ □ . and $y_1 \in f(U), y_2 \in f(V)$ and hence *Y* is totally R-pairwise disconnected space

-:Now we give the following definitions

IV.9 Definitions:- Let $f : X \to Y$ be a mapping from a bitopological space X onto a bitoplogical space Y then

f is called α-pairwise light mapping if the fiber $f^{1}(y)$ for all $y \in Y$ is totally (1 .α-pairwise disconnected set

f is called pre-pairwise light mapping if the fiber $f^{1}(y)$ for all $y \in Y$ is totally (2 .pre-pairwise disconnected set

f is called R-pairwise light mapping if the fiber $f^{1}(y)$ for all $y \in Y$ is totally (3 .R-pairwise disconnected set

f is called sR-pairwise light mapping if the fiber $f^{1}(y)$ for all $y \in Y$ is totally (4 .sR-pairwise disconnected set

f is called s-α-pairwise light mapping if the fiber $f^{1}(y)$ for all $y \in Y$ is totally (5 .s-α-pairwise disconnected set

f is called s-pairwise light mapping if the fiber $f^{1}(y)$ for all $y \in Y$ is totally s-pairwise (6 .disconnected set

-:Now we give the relation between themes

IV.10 Proposition:- Every α-pairwise light mapping is pre-pairwise light mapping

Proof:- Let $f : X \to Y$ be α-pairwise light mapping the for all $y \in Y$, $f^{1}(y)$ is totally α-pairwise disconnected set and since every totally α-pairwise disconnected space is totally pre-pairwise disconnected space then for all $y \in Y$, $f^{1}(y)$ is totally pre-pairwise □ .disconnected set and hence *f* is pre-pairwise light mapping

IV.11 Proposition:- Every α-pairwise light mapping is s-pairwise light mapping

Proof:- Let $f : X \to Y$ be α-pairwise light mapping the for all $y \in Y$, $f^{1}(y)$ is totally α-pairwise disconnected set and since every totally α-pairwise disconnected space is totally s-pairwise disconnected space then for all $y \in Y$, $f^{1}(y)$ is totally s-pairwise □ .disconnected set and hence *f* is s-pairwise light mapping

IV.12 Proposition:- Every α-pairwise light mapping is s-α-pairwise light mapping

Proof:- Let $f : X \rightarrow Y$ be α-pairwise light mapping the for all $y \in Y$, $f^{1}(y)$ is totally α-pairwise disconnected set and since every totally α-pairwise disconnected space is totally s-α-pairwise disconnected space then for all $y \in Y$, $f^{1}(y)$ is totally s-α-pairwise .disconnected set and hence *f* is s-α-pairwise light mapping □

IV.13Proposition:- Every pre-pairwise light mapping is R-pairwise light mapping

Proof:- Let $f : X \rightarrow Y$ be pre-pairwise light mapping the for all $y \in Y$, $f^{1}(y)$ is totally pre-pairwise disconnected set and since every totally pre-pairwise disconnected space is totally R-pairwise disconnected space then for all $y \in Y$, $f^{1}(y)$ is totally R-pairwise .disconnected set and hence *f* is R-pairwise light mapping □

IV.14Proposition:- Every R-pairwise light mapping is sR-pairwise light mapping

Proof:- Let $f : X \rightarrow Y$ be R-pairwise light mapping the for all $y \in Y$, $f^{1}(y)$ is totally R-pairwise disconnected set and since every totally R-pairwise disconnected space is totally sR-pairwise disconnected space then for all $y \in Y$, $f^{1}(y)$ is totally sR-pairwise .disconnected set and hence *f* is sR-pairwise light mapping □

.**IV.15Proposition:-** Every pre-pairwise light mapping is sR-pairwise light mapping

(Proof:- It easy from the propositions (IV.13 and IV.14 □

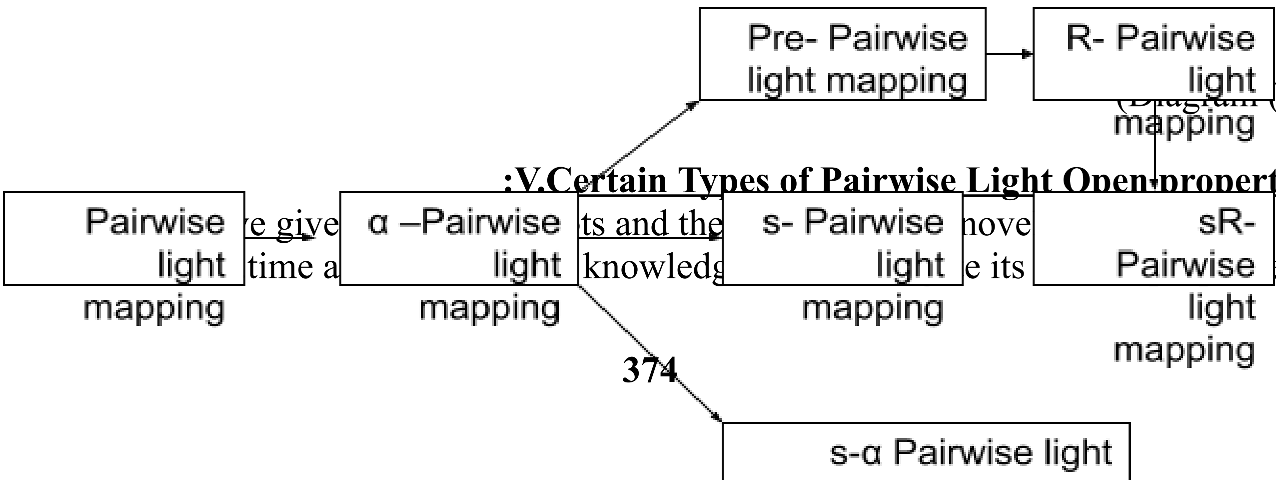
.**IV.16Proposition:-** Every α-pairwise light mapping is R-pairwise light mapping

(Proof:- It easy from the propositions(IV.10 and IV.13 □

.**IV.17Proposition:-** Every α-pairwise light mapping is sR-pairwise light mapping

(Proof:- It easy from the propositions(IV.10 . IV.13 and IV.14 □

As a consequence of propositions (IV.10, IV.11, IV.12, IV.13, IV.14, IV.15, IV.16 and -:IV.17) we have the following diagram



(Diagram (4

:V.Certain Types of Pairwise Light Open property

...e give ... ts and the ... move ... time a ... knowledg ... e its ... es

<u>V.1Definitions:-</u> Let $f : X \to Y$ be a mapping from a bitopological space X onto a bitoplogical space Y then

f is called α-pairwise light open mapping if f is α-pairwise light mapping and (1
. α-open mapping in the same time

f is called pre-pairwise light open mapping if f is pre-pairwise light mapping and (2
. pre-open mapping in the same time

f is called sR-pairwise light open mapping if f is sR-pairwise light mapping and (3
. sR-open mapping in the same time

<u>V.2 Definition:</u> let ***P*** a property from the properties of the topological spaces then ***P*** is
-:called a α- pairwise light open property if satisfy the following
α-pairwise light open mapping keep the P property (i.e if *f:X→Y* be a α-pairwise **-1**
.light open mapping and *X* has the ***P*** property then *Y* also , has it
.(***P*** transfer to the subspace (hereditary property **-2**

<u>V.3 Definition:</u> let ***P*** a property from the properties of the topological spaces then ***P*** is
-:called a pre- pairwise light open property if satisfy the following
pre-pairwise light open mapping keep the P property (i.e if *f:X→Y* be a **-1**
.pre-pairwise light open mapping and *X* has the ***P*** property then *Y* also , has it
.(***P*** transfer to the subspace (hereditary property **-2**

<u>V.4 Definition:</u> let ***P*** a property from the properties of the topological spaces then ***P*** is
-:called a sR- pairwise light open property if satisfy the following
sR-pairwise light open mapping keep the P property (i.e if *f:X→Y* be a sR-pairwise **-1**
.light open mapping and *X* has the ***P*** property then *Y* also , has it
.(***P*** transfer to the subspace (hereditary property **-2**

<u>V.5 Definition:</u> A space *X* is a α- T_1- space iff whenever *x* and *y* are distinct points in *X* , there is two α- open sets *U* and *V* such that
$(a \in U \wedge b \notin U) \wedge (b \in V \wedge a \notin V)$.

<u>V.6 Definition:</u> A space *X* is a α-T_2- space (α-hausdorff space) iff whenever *x* and *y* are distinct points of *X* , there are α- disjoint open sets *U* and *V* such that
$(a \in U \wedge b \in V), U \cap V = \phi$.

<u>V.7 Definition:</u> A space *X* is a α-Regular space iff whenever *F* is α-closed in *X* and
. $F \subset V$ $x \notin F$, then there are α- disjoint open sets U and V with $x \in U$ and

<u>V.8 Remark:</u> It is clear that the property of α-T_1 space , α-T_2 space and α-Regular
.space is a hereditary property

<u>V.9Theorem:</u> let *f:X→Y* be a α-pairwise light open mapping. then the following
. properties is a α-pairwise light open properties
.α- T_1 space **-1**
.α- T_2 space **-2**
.α- Regular space **-3**

Proof: **1**-suppose that X is a α- T_1 space and to prove that Y is also α- T_1 space. Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f is a α-pairwise light then f is a surjective and hence there exists $x_1, x_2 \in X$ such that $f(x_1)=y_1$ and $f(x_2)=y_2$ and since X is a α-T_1 space then there exists two α-open sets U_1 *and* U_2 in X such that $(x_1 \in U_1 \wedge x_2 \notin U_1) \wedge (x_2 \in U_2 \wedge x_1 \notin U_2)$ and since f is α-open mapping then each of $f(U_1)$ and $f(U_2)$ are α-open sets in Y such that $(y_1 \in f(U_1) \wedge y_2 \in f(U_2)) \wedge (y_2 \notin f(U_1) \wedge y_1 \notin f(U_2))$ and hence Y is a α-T_1 space and since the property of α-T_1 space is a hereditary property then α-T_1 space is a α- pairwise light open property □

2- Suppose that X is a α-T_2 space and to prove that Y is also α-T_2 space. Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$ .Since f is a surjective mapping and hence there exists $x_1, x_2 \in X$ such that $f(x_1)=y_1$ and $f(x_2)=y_2$ and since X is a α-T_2 space then there exists two α-disjoint open sets U_1 *and* U_2 in X such that $(x_1 \in U_1 \wedge x_2 \in U_2) \wedge U_1 \cap U_2$ and since f is a α-open mapping then each of $f(U_1)$ and $f(U_2)$ are α-open sets in Y such that $(y_1 = f(x_1) \in f(U_1) \wedge y_2 = f(x_2) \in f(U_2)) \wedge f(U_1) \cap f(U_2) = \phi$ and hence Y is a α-T_2 space and since the property of α-T_2 space is a hereditary property then α-T_2 space is a α-pairwise light open property □

3-Suppose that X is a α-regular space and to prove that Y is a α-regular space. Let $y \in Y$ and F is a α-closed set in Y such that $y \notin F$ and since f is α-pairwise light mapping then f is a surjective mapping then there exists $x \in X \ni f(x) = y \wedge x \notin f(F)$ and since f is α-continuous then $f^{-1}(F)$ is α-closed in X and since X is a α-regular space then there exists two α-open sets U_1 and U_2 in X such that $(x \in U_1 \wedge f^{-1}(F) \subset U_2) \wedge U_1 \cap U_2 = \phi$ and since f is a α-open mapping then each of $f(U_1)$ and $f(U_2)$ are α-open sets in Y such that $F \subseteq f(f^{-1}(F)) \subset f(U_2) \wedge y = f(x) \in f(U_1) \wedge f(U_1) \cap f(U_2) = \phi$ and hence Y is a α-regular space and since the property of α-Regular space is a hereditary property then α-Regular is a α-pairwise light open property □

<u>V.10Definition[4]:</u> A space X is a pre- T_1- space iff whenever x and y are distinct points in X, there is two pre- open sets U and V such that $(a \in U \wedge b \notin U) \wedge (b \in V \wedge a \notin V)$.

<u>V.11 Definition[4]:</u> A space X is a pre-T_2- space (pre-hausdorff space) iff whenever x and y are distinct points of X, there are pre- disjoint open sets U and V such that $(a \in U \wedge b \in V), U \cap V = \phi$.

<u>V.12 Definition:</u> A space X is a pre-Regular space iff whenever F is pre-closed in X and $x \notin F$, then there are pre- disjoint open sets U and V with $x \in U$ and $F \subset V$.

<u>V.13 Remark:</u> It is clear that the property of pre-T_1 space , pre-T_2 space and .pre-Regular space is a hereditary property

<u>V.14Theorem:</u> let $f:X\rightarrow Y$ be a pre-pairwise light open mapping. then the following . properties is a pre-pairwise light open properties
.pre- T_1 space -**1**
.pre- T_2 space -**2**
.pre- Regular space -**3**

Proof: **1**-suppose that X is a pre- T_1 space and to prove that Y is also pre- T_1 space. Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f is a pre-pairwise light then f is a surjective and hence there exists $x_1, x_2 \in X$ such that $f(x_1)=y_1$ and $f(x_2)=y_2$ and since X is a pre-T_1 space then there exists two pre-open sets U_1 *and* U_2 in X such that $(x_1 \in U_1 \wedge x_2 \notin U_1) \wedge (x_2 \in U_2 \wedge x_1 \notin U_2)$ and since f is pre-open mapping then each of $f(U_1)$ and $f(U_2)$ are pre-open sets in Y such that and hence Y is a pre-T_1 space and since the $(y_1 \in f(U_1) \wedge y_2 \in f(U_2)) \wedge (y_2 \notin f(U_1) \wedge y_1 \notin f(U_2))$ property of pre-T_1 space is a hereditary property then pre-T_1 space is a pre- pairwise □ light open property

Suppose that X is a pre-T_2 space and to prove that Y is also pre-T_2 space. Let -**2** $y_1, y_2 \in Y$ such that $y_1 \neq y_2$ .Since f is a surjective mapping and hence there exists $x_1, x_2 \in X$ such that $f(x_1)=y_1$ and $f(x_2)=y_2$ and since X is a pre-T_2 space then there exists two α-disjoint open sets U_1 *and* U_2 in X such that $(x_1 \in U_1 \wedge x_2 \in U_2) \wedge U_1 \cap U_2$ and since f is a pre-open mapping then each of $f(U_1)$ and $f(U_2)$ are pre-open sets in Y such that $(y_1 = f(x_1) \in f(U_1) \wedge y_2 = f(x_2) \in f(U_2)) \wedge f(U_1) \cap f(U_2) = \phi$ and hence Y is a pre-T_2 space and since the property of pre-T_2 space is a hereditary property then pre-T_2 space is a □ pre-pairwise light open property

Suppose that X is a pre-regular space and to prove that Y is a pre-regular space. Let-**3** $y \in Y$ and F is a pre-closed set in Y such that $y \notin F$ and since f is pre-pairwise light mapping then f is a surjective mapping then there exists $x \in X \ni f(x) = y \wedge x \notin f(F)$ and since f is pre-continuous then $f^{-1}(F)$ is pre-closed in X and since X is a pre-regular space then there exists two pre-open sets U_1 and U_2 in X such that $(x \in U_1 \wedge f^{-1}(F) \subset U_2) \wedge U_1 \cap U_2 = \phi$ and since f is a pre-open mapping then each of $f(U_1)$ and $f(U_2)$ are pre-open sets in Y such that $F \subseteq f(f^{-1}(F)) \subset f(U_2) \wedge y = f(x) \in f(U_1) \wedge f(U_1) \cap f(U_2) = \phi$ and hence Y is a pre-regular space and since the property of pre-Regular space is a hereditary property then pre- □ Regular is a pre-pairwise light open property

<u>V.15Definition[8]:</u> A space X is a sR- T_1- space iff whenever x and y are distinct points in X , there is two sR- open sets U and V such that

$(a \in U \wedge b \notin U) \wedge (b \in V \wedge a \notin V)$.

V.16 Definition[8]: A space X is a sR-T_2- space (s-hausdorff space) iff whenever x and y are distinct points of X, there are sR- disjoint open sets U and V such that $(a \in U \wedge b \in V), U \cap V = \phi$.

V.17 Definition: A space X is a sR- space iff whenever F is pre-closed in X and $x \notin F$, then there are sR- disjoint open sets U and V with $x \in U$ and $F \subset V$.

V.18 Remark: It is clear that the property of sR-T_1 space , sR-T_2 space and sR-space is a hereditary property.

V.19Theorem: let f:$X \to Y$ be a sR-pairwise light open mapping. then the following properties is a sR -pairwise light open properties.

1- sR - T_1 space.

2- sR - T_2 space.

3- sR -space.

Proof: **1**-suppose that X is a sR - T_1 space and to prove that Y is also sR - T_1 space. Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f is a sR -pairwise light then f is a surjective and hence there exists $x_1, x_2 \in X$ such that $f(x_1)=y_1$ and $f(x_2)=y_2$ and since X is a sR -T_1 space then there exists two sR -open sets U_1 *and* U_2 in X such that $(x_1 \in U_1 \wedge x_2 \notin U_1) \wedge (x_2 \in U_2 \wedge x_1 \notin U_2)$ and since f is sR -open mapping then each of $f(U_1)$ and $f(U_2)$ are sR -open sets in Y such that $(y_1 \in f(U_1) \wedge y_2 \in f(U_2)) \wedge (y_2 \notin f(U_1) \wedge y_1 \notin f(U_2))$ and hence Y is a sR -T_1 space and since the property of sR -T_1 space is a hereditary property then sR -T_1 space is a sR -pairwise light open property □

2- Suppose that X is a sR -T_2 space and to prove that Y is also sR -T_2 space. Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$ .Since f is a surjective mapping and hence there exists $x_1, x_2 \in X$ such that $f(x_1)=y_1$ and $f(x_2)=y_2$ and since X is a sR -T_2 space then there exists two sR -disjoint open sets U_1 *and* U_2 in X such that $(x_1 \in U_1 \wedge x_2 \in U_2) \wedge U_1 \cap U_2$ and since f is a sR -open mapping then each of $f(U_1)$ and $f(U_2)$ are sR -open sets in Y such that $(y_1 = f(x_1) \in f(U_1) \wedge y_2 = f(x_2) \in f(U_2)) \wedge f(U_1) \cap f(U_2) = \phi$ and hence Y is a sR -T_2 space and since the property of sR -T_2 space is a hereditary property then sR -T_2 space is a sR -pairwise light open property □

3-Suppose that X is a sR - space and to prove that Y is a sR - space. Let $y \in Y$ and F is a sR -closed set in Y such that $y \notin F$ and since f is sR -pairwise light mapping then f is a surjective mapping then there exists $x \in X \ni f(x) = y \wedge x \notin f(F)$ and since f is continuous then $f^{-1}(F)$ is sR -closed in X and since X is a sR - space then there exists two sR -open sets U_1 and U_2 in X such that $(x \in U_1 \wedge f^{-1}(F) \subset U_2) \wedge U_1 \cap U_2 = \phi$ and since f

is a sR -open mapping then each of $f(U_1)$ and $f(U_2)$ are sR-open sets in Y such that $F \subseteq f(f^{-1}(F)) \subset f(U_2) \wedge y = f(x) \in f(U_1) \wedge f(U_1) \cap f(U_2) = \phi$ and hence Y is a sR -regular space and since the property of sR - space is a hereditary property then sR -space is a sR -pairwise light open property □

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