On Weakly ij-Open Functions In Bitopological Spaces
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:Abstract
In this work, we study new characterizations of weakly ij-open function. Also we study the relation between the weakly ij-open function and ij- contra-closed function and proved that if f is a ij- contra- open function and ij-pre - open ,then f is i-open .function

Introduction: Kelly in 1963 [2], defined a set equipped with two topologies is called -1 a bitopological spaces denoted by (X, τ_1, τ_2) where (X, τ_1) and (X, τ_2) are two topological spaces . The concept of almost –s-continuous functions introduced by Noiri and Khan in [4] and the concept of weakly continuous functions was introduced by Levine in [3].In this paper we proved some properties of weakly ij- open functions . Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X the closure of A and the interior of A with respect to τ_i are denoted by $\overline{(A)}^{i}$ and $(A)^{\circ i}$ respectively for i=1,2

:Preliminaries -2

Definitions(2-1)[1]: A subset A of a bitopological space X is said to be ij-semi open .set if $A \subset \overline{((A^{\circ i}))}^{j}$ for i≠j and i,j=1,2

Definition(2-2)[5]: A subset A of a bitopological space X is said to be ij- α-open set if . $A \subset ((\overline{A^{\circ i}})^{j})^{\circ i}$ for i≠j and i,j=1,2

Definition(2-3)[5] : A subset A of a bitopological space X is said to be ij- pre-open set .if $A \subset ((\overline{(A)}^{j})^{\circ i}$ for i≠j and i,j=1,2

Now we give the following definitions

Definition (2-4): Let f: $(X, \tau_1, \tau_2) \longrightarrow (X, \sigma_1, \sigma_2)$ be a function then f is said to be .ij-semi open if for each τ_i-open set U of X , f(U) is ij- semi open set in Y

Definition (2-5): Let f: $(X, \tau_1, \tau_2) \longrightarrow (X, \sigma_1, \sigma_2)$ be a function then f is said to be .ij-pre- open if for each τ_i-open set U of X , f(U) is ij- pre- open set in Y

Definition (2-6): Let f: $(X, \tau_1, \tau_2) \longrightarrow (X, \sigma_1, \sigma_2)$ be a function then f is said to be . $f(U) \subset ((\overline{(f(U))}^{j})^{\circ i}$, weakly ij- open if for each τ_i-open set U of X

Definition (2-7)[4]: Let f: $(X, \tau_1, \tau_2) \longrightarrow (X, \sigma_1, \sigma_2)$ be a function then f is said to be .ij- contra closed if f(U) is σ_i- open in Y for every τ_j-closed set U in X

Definition (2-8)[4]: Let f: $(X, \tau_1, \tau_2) \longrightarrow (X, \sigma_1, \sigma_2)$ be a function then f is said to be .ij- contra open if f(U) is σ_j- closed in Y for every τ_i-closed set U in X

:Some New Characterizations of Weakly ij-Open Function -3

:Now we give the following propositions

Proposition (3-1): Let f: $(X, \tau_1, \tau_2) \longrightarrow (X, \sigma_1, \sigma_2)$ be a function then the following : properties are equivalent

a) f is weakly ij-open function

. $\overline{(f(U^{\circ i})}^{j} \subset f(U)$, b) for every τ_i- closed set U in X

$\overline{(f(U)}^i \subset f(\overline{U}^i)$, c) for every τ_i- open set U in X.

Proof: (a)⟹ (b) Suppose that U is τ_i- closed set in X then X-U is τ_i- open set in X and hence $X - f(U) = f(X-U) \subset (f(\overline{X-U})^j)^{\circ i} = (f(X-(U)^{\circ i})^{\circ i} = (Y - f(U^{\circ i}))^{\circ i} = Y - \overline{(f(U^{\circ j})}^i$ and this implies that $\overline{(f(U^{\circ j})}^i \subset f(U)$.

⟹ b) (c) Suppose that U be any τ_i- open set in X then from (b) we get)

$= \overline{(f(U)}^i \subset (f(((\overline{U})^{\circ j})}^i \subset \overline{(f(U^{\circ j}))}^i \subset f(\overline{U}^i)$ and hence $\overline{(f(U)}^i \subset f(\overline{U}^i)$

⟹ c) (a) Suppose that U be any τ_i- open set in X then, we have)

$= Y - (f(\overline{U}^j)^{\circ i} = \overline{(Y - f(\overline{U}^j))}^i = f(X - U^{\circ i}) \subset f(X - ((\overline{U})^j)^{\circ i} = \overline{f((X-(\overline{U})^j)}^i \subset \overline{(f(X-(\overline{U}^j))}^i$

$f(X-U) = Y - f(U)$ .therefore f is weakly ij-open $f(U) \subset (f(\overline{U}^j))^{\circ i}$ .and hence

Proposition (3-2): Let f: (X ,τ_1 ,τ_2) ⟶ (X, σ_1, σ_2) be a function if for every ij-α-open set U, $f(U) \subset (f(\overline{U}^j))^{\circ i}$ then f is weakly ij-open.

Proof: Suppose that U is τ_i- open set in X then U is ij- α-open set in X and hence from definition of ij- α-open set we get $f(U) \subset (f(\overline{U}^j))^{\circ i}$ and hence f is weakly ij-open function.

Proposition (3-3): Let f: (X ,τ_1 ,τ_2) ⟶ (X, σ_1, σ_2) be a function if f is weakly ij-open function then for every τ_j- closed set U in X $f(U^{\circ i}) \subset (f(U))^{\circ i}$.

Proof: Suppose that f is weakly ij-open function and let U is τ_j- closed set U in X then $U^{\circ i}$ τ_i- open set in X and since f is weakly ij-open function then $f(U^{\circ i}) \subset (f(\overline{(U^{\circ i})}))^{\circ i} \subset (f(U))^{\circ i}$.

Proposition (3-4): Every ij – contra - closed function is weakly ij – open.

Proof: Let Let f: (X ,τ_1 ,τ_2) ⟶ (X, σ_1, σ_2) be a ij- contra closed function and let U be any τ_i- open set in X and hence $\overline{U}^j$ is τ_i- closed set in X, then $f(U) \subset f(\overline{U}^j) \subset (f(\overline{U}^j))^{\circ i}$ then from definition (2-6) f is weakly ij- open function.

Proposition(3-5): Let f: (X ,τ_1 ,τ_2) ⟶ (X, σ_1, σ_2) be a ij- contra- open function and ij-pre - open ,then f is i-open function.

Proof:Let U be any τ_i- open set in X, since f is ij- pre-open then from definition (2-5) $f(U) \subset ((\overline{f(U)}^{j})^{\circ})^{i}$ and since f is ij- contra-open function then f(U) is σ_j- closed set in Y , therefore $f(U) \subset ((\overline{f(U)}^{j})^{\circ})^{i} = (f(U))^{\circ i}$, hence f(U) is σ_i- open in Y and f is i-open .function

Corollary(3-6):Let U be any ij-pre-open and $f(U) \subset (f(\overline{U}^{j}))^{\circ i}$, then $f(U) \subset (f(\overline{U}^{j}))^{\circ i}$. for every ij-α-open set in X

.Proof: since every ij- α-open set is ij-pre-open set then we get the result

Corollary(3-7): Let f: $(X, \tau_1, \tau_2) \longrightarrow (X, \sigma_1, \sigma_2)$ be a function and Let U be any .ij-pre-open , $f(U) \subset (f(\overline{U}^{j}))^{\circ i}$,then f is weakly ij-open function

.(Proof: It is easy from the corollary(3-6) and proposition(3-2

Proposition(3-8): Let f: $(X, \tau_1, \tau_2) \longrightarrow (X, \sigma_1, \sigma_2)$ be a ij-semi-open function , then . $((\overline{f(U)}^{i})^{\circ})^{j} \subset f(U)$,for every τ_i- closed set U in X

Proof: Let U be any τ_i- closed set U in X then $X-U$ is τ_i- open set U in X and since f is ij-semi-open function then $f(X-U) \subset \overline{((f(X-U))^{\circ i})}^{j}$, hence $Y - fU) =$ $f(X-U) \subset \overline{((f(X-U))^{\circ i})}^{j} = \overline{((Y-f(U))^{\circ i})}^{j} = Y - ((\overline{f(U)}^{i})^{\circ})^{j}$. , hence . $((\overline{f(U)}^{i})^{\circ})^{j} \subset f(U)$

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