Some new Types of closed sets in Bitopological space

Azal Ja'far Moosa
University of Babylon, College of Education.

الخلاصة

في هذه البحث، نُقدّمُ وندْرسُ بعض الأنواع من المجموعات الجزئية المُغلقة المُعمّمة مِنَ الفضاء التبولوجي، كما سَنُلخّصُ العلاقاتَ بينها مع براهينها. كما سنُقدّمُ وندْرسُ أنواعَ جديدةَ مِنْ الدوالِ المستمرة عليها، والعلاقات بينها مع براهينها. كما سنثبت مجموعة من القضايا المقترحة على تلك المفاهيم.

Abstract

In this paper, we introduce and study types a new of closed set called gs^* -closed set, sg^* -closed set, $g\alpha^*$ -closed set, αg^* -closed set of Bitopological space, and we will summarize the relationships between them and we shall prove every pointed on it. Also we will introduce and study some types of continuous functions on it, also, we shall summarize the relationships between them, and .proved every pointed on it. Several properties of these concepts are proved

Key words: closed set , g -closed set, α -closed set, $semi$ -closed set.

Introduction .1

A bitopological space (X,τ_1,τ_2) [1] isanon empty set with two topologies τ_1 and τ_2 on X. In 1970, Levine [6] introduced the concept of generalized closed sets in a topological space, shortly (g -closed) where he defined a subset A of a topological space X to be g -closed if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open. In 2009,FalahIn [5]studied a new types of g -closed subsets of topological spaces, which is called gs^* -closed set, sg^* -closed set, $g\alpha^*$ -closed set, αg^* -closed set. Some types of g -closed subsets of Bitopological spaces is studied in this paper which is denoted gs^* -closed set, sg^* -closed set, $g\alpha^*$ -closed set, αg^* -closed set.

2. Preliminaries

Throughout this paper, for a subset A of a Bitopological space (X,τ_1,τ_2) $cl_{\tau_{i=1,2}}(A)$ (resp. $\operatorname{int}_{\tau_{i=1,2}}(A)$, $scl_{\tau_{i=1,2}}(A)$, $\alpha cl_{\tau_{i=1,2}}(A)$) will denote to the closure (resp. interior, smallest $semi$ -closed set containing A, smallest α -closed set containing A), also the symbol will indicate the end of a proof.

For the sake of convenience, we begin with some basic concepts, although most of these concepts can be found from the references of this paper.

Definition (2.1) [4] A subset A of a Bitopological space (X,τ_1,τ_2) is called:

1. $semi$ -closed if $\operatorname{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq A$,
2. α -closed if $cl_{\tau_i}(\operatorname{int}_{\tau_i}(cl_{\tau_i}(A)))\subseteq A$, i=1,2 .

Definition (2.2) [4] The complement of $semi$ -closed (resp. α -closed) is called $\tau_i - semi$ -open,i=1or 2 (resp. $\tau_i - \alpha$ - open,i=1or 2).

Definition (2.3) A subset A of a Bitopological space (X,τ_1,τ_2) is called:

1.generalized closed (g-closed) set [5] if $cl_{\tau_{i=1,2}}(A)\subseteq U$ whenever $A\subseteq U$ and U is τ_i- open set,i=1or2,

2.generalized $semi$-closed (gs-closed) set [7] if $scl_{\tau_{i=1,2}}(A)\subseteq U$ whenever $A\subseteq U$ and U is τ_i- open set ,i=1or2,

3. $semi$-generalized closed (sg-closed) set [6] if $scl_{\tau_{i=1,2}}(A)\subseteq U$ whenever $A\subseteq U$ and U is τ_i- $semi$-open set ,i=1or2,

4.generalized α-closed ($g\alpha$-closed) set [2] if $\alpha cl_{\tau_{i=1,2}}(A)\subseteq U$ whenever $A\subseteq U$ and U is $\tau_i-\alpha$-open set ,i=1or 2,

5. α-generalized closed (αg-closed) set [1] if $\alpha cl_{\tau_{i=1,2}}(A)\subseteq U$ whenever $A\subseteq U$ and U is τ_i- open set,i=1or2.

Definition (2.4) The complement of g-closed (resp. gs-closed, sg-closed, $g\alpha$-closed, αg-closed) is called g-open (resp. gs-open, sg- open, $g\alpha$- open, αg-open)

Remark (2.5)

The relationships between the concepts in definitions (2.1) and (2.3) summarized in the following diagram:

Diagram (2-1)

Now, we will prove every pointed in the above diagram in the following propositions

Proposition (2.6)

Every closed subset of a Bitopological space (X,τ_1,τ_2) is g-closed.

Proof:

Let $A\subseteq X$ be closed set, and let $A\subseteq U$, where U is τ_i- open set, i=1,2, since A is closed set then $cl_{\tau_{i=1,2}}(A)=A$, hence $cl_{\tau_{i=1,2}}(A)\subseteq U$, i.e. A is g-closed.

Proposition (2.7)

Every g-closed subset of a Bitopological space (X,τ_1,τ_2) is αg-closed.

Proof:

Let $A\subseteq X$ be g-closed set, and let $A\subseteq U$, where U is τ_i- open set,i=1,2, since A is g-closed set then $cl_{\tau_{i=1,2}}(A)\subseteq U$, and hence $\operatorname{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq \operatorname{int}_{\tau_i}(U)$, i=1,2, but U is open set, so $\operatorname{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq U$,i=1,2. Since $\alpha cl_{\tau_{i=1,2}}(A)$ is the smallest α-closed set containing A, so, $\alpha cl_{\tau_i}(A)=A\cup cl_{\tau_i}(\operatorname{int}_{\tau_i}(cl_{\tau_i}(A)))\subseteq A\cup cl_{\tau_i}(U)\subseteq U$, i=1,2. i.e. A is αg-closed.

Proposition (2.8)

Every closed subset of a Bitopological space (X,τ_1,τ_2) is α -closed.

Proof:

Let $A\subseteq X$ be τ_i- closed, set i=1,2, then $cl_{\tau_i}(A)=A$, hence $\text{int}_{\tau_i}(cl_{\tau_i}(A))=\text{int}_{\tau_i}(A)$,i=1,2, but $\text{int}_{\tau_{i=1,2}}(A)\subseteq A$, so $\text{int}_{\tau_{i=1,2}}(cl_{\tau_{i=1,2}}(A))\subseteq U$, and $cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))\subseteq cl_{\tau_i}(A)$, i=1,2. Then $cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))\subseteq A$,i=1,2, i.e. A is α -closed.

Proposition (2.9)

Every α -closed subset of a Bitopological space (X,τ_1,τ_2) is $g\alpha$ -closed.

Proof:

Let $A\subseteq X$ be $\tau_i-\alpha$ -closed set, and let $A\subseteq U$, where U is α -open set, since A is $\tau_i-\alpha$ -closed set, then $cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))\subseteq A\subseteq U$,i=1,2, since $\alpha cl_{\tau_{i=1,2}}(A)$ is the smallest α -closed set containing A , so,

$$\alpha cl_{\tau_i}(A)=A\cup cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))$$
$$\subseteq A\cup U$$
$$\subseteq U\ ,\ i=1,2.$$

i.e. A is $g\alpha$ -closed.

Proposition (2.10)

Every $g\alpha$ -closed subset of a Bitopological space (X,τ_1,τ_2) is gs -closed.

Proof:

Let $A\subseteq X$ be $\tau_i-g\alpha$ -closed set,i=1,2, and let $A\subseteq U$, where U is τ_i- open set, since A is $\tau_i-g\alpha$ -closed set,i=1,2 then $\alpha cl_{\tau_{i=1,2}}(A)\subseteq U$ and since $\alpha cl_{\tau_i}(A)=A\cup cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))$,i=1,2, then $cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))\subseteq \alpha cl_{\tau_i}(A)\subseteq U$,i=1,2, but $\text{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))$,i=1,2then $\text{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq U$,i=1,2. Since $cl_s(A)$ is the smallest $semi$ -closed set containing A , so,

$$scl_{\tau_i}(A)=A\cup \text{int}_{\tau_i}(cl_{\tau_i}(A))$$
$$\subseteq U, i=1,2.$$

i.e. A is gs -closed.

Proposition (2.11)

Every g -closed subset of a Bitopological space (X,τ_1,τ_2) is gs -closed.

Proof:

Let $A\subseteq X$ be τ_i-g -closed set, and let $A\subseteq U$, where U is τ_i- open set, since A is g -closed set then $cl_{\tau_i}(A)\subseteq U$, and hence $\text{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq \text{int}_{\tau_i}(U),\ i=1,2$, but U is τ_i- open set, so $\text{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq U,\ i=1,2,$. Since $scl_{\tau_{i=1,2}}(A)$ is the smallest $semi$ -closed set containing A , so,

$scl_{\tau i}(A) = A \cup \operatorname{int}_{\tau i}(cl_{\tau i}(A))$
$\subseteq U, i = 1,2$,

i.e. A is gs -closed.

Proposition (2.12)

Every $g\alpha$ -closed subset of a Bitopological space (X,τ_1,τ_2) is αg -closed.

Proof:

Let $A \subseteq X$ be $\tau_i -$ $g\alpha$ -closed set, and let $A \subseteq U$, where U is $\tau_i -$ open set, since A is $g\alpha$ -closed set, then $\alpha cl_{\tau i, i=1,2}(A) \subseteq U$, i.e. A is αg -closed.

Proposition (2.13)

Every αg -closed subset of a Bitopological space (X,τ_1,τ_2) is gs -closed.

Proof:

Let $A \subseteq X$ be $\tau_i -$ αg -closed set, and let $A \subseteq U$, where U is $\tau_i -$ open set, since A is $\tau_i -$ αg -closed set, then $\alpha cl_{\tau i, i=1,2}(A) \subseteq U$, and since $\alpha cl_{\tau i}(A) = A \cup cl_{\tau i}(\operatorname{int}_{\tau i}(cl_{\tau i}(A)))$,i=1,2, then $cl_{\tau i}(\operatorname{int}_{\tau i}(cl_{\tau i}(A))) \subseteq \alpha cl_{\tau i}(A) \subseteq U$,i=1,2, but $\operatorname{int}_{\tau i}(cl_{\tau i}(A)) \subseteq cl_{\tau i}(\operatorname{int}_{\tau i}(cl_{\tau i}(A)))$,i=1,2 then $\operatorname{int}_{\tau i}(cl_{\tau i}(A)) \subseteq U$,i=1,2. Since $scl_{\tau i, i=1,2}(A)$ is the smallest $semi$ -closed set containing A , so,
$scl_{\tau i}(A) = A \cup \operatorname{int}_{\tau i}(cl_{\tau i}(A))$
$\subseteq U, i = 1,2,$

i.e. A is gs -closed.

Proposition (2.14)

Every α -closed subset of a Bitopological space (X,τ_1,τ_2) is $semi$ -closed.

Proof:

Let $A \subseteq X$ be $\tau_i -$ α -closed set, then $cl_{\tau i}(\operatorname{int}_{\tau i}(cl_{\tau i}(A))) \subseteq A,\ i = 1,2,$ since $\operatorname{int}_{\tau i}(cl_{\tau i}(A)) \subseteq cl_{\tau i}(\operatorname{int}_{\tau i}(cl_{\tau i}(A)))$,i=1,2, so $\operatorname{int}_{\tau i, i=1,2}(cl_{\tau i, i=1,2}(A)) \subseteq A$, i.e. A is $semi$ -closed.

Proposition (2.15)

Every $semi$ -closed subset of a Bitopological space (X,τ_1,τ_2) is sg -closed.

Proof:

Let $A \subseteq X$ be $\tau_i -$ $semi$ -closed set, and let $A \subseteq U$, where U is $\tau_i -$ $semi$ -open set, since A is $semi$ -closed set, then $\operatorname{int}_{\tau i}(cl_{\tau i}(A)) \subseteq A \subseteq U$,i=1,2. Since $scl_{\tau i, i=1,2}(A)$ is the smallest $semi$ -closed set containing A , so,
$scl_{\tau i}(A) = A \cup \operatorname{int}_{\tau i}(cl_{\tau i}(A))$
$\subseteq U, i = 1,2$,

i.e. A is sg -closed.

Proposition (2.16)

Every sg -closed subset of a Bitopological space (X,τ_1,τ_2) is gs -closed.

Proof:

Let $A \subseteq X$ be $\tau_i - sg$ -closed set, and let $A \subseteq U$, where U is $\tau_i -$ open set, since A is sg -closed set, then $scl_{\tau(i=1,2)}(A) \subseteq U$, i.e. A is closed.

Now, we will give some example to show that the inverse pointed in the diagram (2.1) is not true

Example (2.17) g -closed set $\nrightarrow$ closed set.

Let $X=\{a,b,c\}$, $\tau_1=\{X,\phi,\{a\},\{c\},\{a,c\}\}$, $\tau_2=\{X,\phi,\{a\}\}$ so $\tau^c=\{\phi,X,\{b,c\},\{a,b\},\{b\}\}$, let $A=\{a\}$, $U=\{X\}$ open set.

Now, since $cl(A)=\{a,b\} \subseteq U$, i.e. $A=\{a\}$ is g -closed set, but it is not closed set.

Example (2.18) α -closed set $\nrightarrow$ $\tau_i -$ closed set, i=1 or2

Let $X=\{a,b,c,d\}$, $\tau_1=\{X,\phi,\{a\},\{c\},\{a,c\},\{a,b,d\}\}$, $\tau_2=\{X,\phi,\{a,c\}\}$ so $\tau^c=\{\phi,X,\{b,c,d\},\{a,b,d\},\{b,d\},\{c\}\}$, let $A=\{b,c\}$.

Now, since $cl(A)=\{b,c,d\} \subseteq U$, and $int(cl(A))=\{c\}$, $cl(int(cl(A)))=\{c\} \subseteq A$ i.e. $A=\{b,c\}$ is α -closed set, but it is not $\tau_i -$ closed set, i=1 or2.

Example (2.19) αg -closed set $\nrightarrow$ g -closed set.

Let $X=\{a,b,c\}$, $\tau_1=\{X,\phi,\{a\},\{c\},\{a,c\}\}$, $\tau_2=\{X,\phi,\{b\}\}$ so $\tau^c=\{\phi,X,\{b,c\},\{a,b\},\{b\}\}$, let $A=\{c\}$,, $U=X$ open set.

Now, since $cl(A)=\{b,c\} \subseteq U$, and $int(cl(A))=\{c\}$, $cl(int(cl(A)))=\{b,c\} \subseteq U$ i.e. $A=\{c\}$ is αg -closed set, but it is not g -closed set, Since if we take $U=\{a\}$, $cl(A)=\{a,b\} \nsubseteq A$.

Example (2.20) gs -closed set $\nrightarrow$ g -closed set.

Let $X=\{a,b,c\}$, $\tau_1=\{X,\phi,\{a\},\{c\},\{a,c\}\}$, so $\tau_2=\{X,\phi,\{b\}\}$, let $A=\{c\}$,, $U=X$ open set.

Now, by proposition (2.10) we have $A=\{c\}$ is gs -closed, but it is not g -closed set.

Example (2.21) gs -closed set $\nrightarrow$ αg -closed set.

Let $X=\{a,b,c\}$, $\tau_1=\{X,\phi,\{a\},\{b\},\{a,b\}\}$, so $\tau_2=\{X,\phi,\{a\}\}$, let $A=\{b\}$,, $U=X$ open set.

Now, since $cl(A)=\{b,c\}\subseteq U$, and $int(cl(A))=\{b\}$, so $cl_s(A)=\{b,c\}\subseteq U$ i.e. $A=\{b\}$ is gs-closed set, but it is not αg-closed set, Since if we take $U=\{a\}$, $cl(A)=\{c,b\}$, and $int(cl(A))=\{b\}$, $cl(int(cl(A)))=\{c,b\}$, so $cl_\alpha(A)=\{c,b\}\not\subseteq U$.

3. (gs^* , sg^* , $g\alpha^*$ and αg^*)-Closed Sets and Continuous Functions on it

In this section we will introduce and investigate new types of g-closed subsets of Bitopological space, which we called it gs^*-closed set, sg^*-closed set, $g\alpha^*$-closed set, αg^*-closed set, the relationships between them are summarized in the diagram (3-1), and we will proved every pointed between them, also, we will give examples for these concepts. Finally we will define new types of continuous functions on our new concepts, also the relationships between them are summarized in the diagram (3-2), and we will proved every pointed between them. We well prove several propositions about these concepts.

Definition (3.1) A subset A of a Bitopological space (X,τ_1,τ_2) is called:

1. gs^*-closed set if $scl_{\tau_{i=1,2}}(A)\subseteq U$ whenever $A\subseteq U$ and U is $\tau_i - gs$-open set , i=1 or2,
2. sg^*-closed set if $scl_{\tau_{i=1,2}}(A)\subseteq U$ whenever $A\subseteq U$ and U is $\tau_i - sg$-open set , i=1 or 2,
3. $g\alpha^*$-closed set if $\alpha cl_{\tau_{i=1,2}}(A)\subseteq U$ whenever $A\subseteq U$ and U is $\tau_i - g\alpha$-open set , i=1 or 2 ,
4. αg^*-closed set if $\alpha cl_{\tau_{i=1,2}}(A)\subseteq U$ whenever $A\subseteq U$ and U is $\tau_i - \alpha g$-open set , i=1 or 2.

Remark (3.2)

The relationships between the concepts in definition (3.1) summarized in the following diagram:

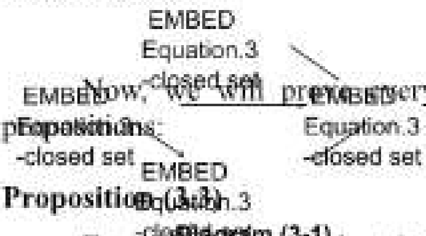

Diagram (3-1)

Now, we will prove every pointed in the above diagram in the following propositions:

Proposition (3.3)

Every sg^*-closed subset,i=1 or2, of a bitopological space (X,τ_1,τ_2) is gs^*-closed.

Proof:

Let $A\subseteq X$ be sg^*-closed set, then $cl_s(A)\subseteq U$ where $A\subseteq U$ and U is sg-open, then U^c is sg-closed, hence U^c is $\tau_i - gs$-closed, then U is $\tau_i - gs$-open and $cl_s(A)\subseteq U$, i.e. A is gs^*-closed.

Proposition (3.4)

Every $g\alpha^*$ -closed subset of a Bitopological space (X,τ_1,τ_2) is gs^* -closed.

Proof:

Let $A\subseteq X$ be $\tau_i-g\alpha^*$ -closed set i=1 or 2, then $scl_{\tau_{i=1,2}}(A)\subseteq U$ where $A\subseteq U$ and U is $\tau_i-g\alpha$ -open set, since $\alpha cl_{\tau_i}(A)=A\cup cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))$,i=1,2, for any $A\subseteq X$ then $cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))\subseteq \alpha cl_{\tau_i}(A)\subseteq U$,i=1,2, but $\text{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))$,i=1,2 then $\text{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq U$,i=1,2. Since $scl_{\tau_{i=1,2}}(A)$ is the smallest $semi$ -closed set containing A, so, $scl_{\tau_i}(A)=A\cup\text{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq U$, i=1,2, and since U is $g\alpha$ -open, then U^c is $g\alpha$ -closed, hence U^c is gs -closed, then U is τ_i-gs -open and $scl_{\tau_{i=1,2}}(A)\subseteq U$, i.e. A is gs^* -closed.

Proposition (3.5)

Every $g\alpha^*$ -closed subset of a Bitopological space (X,τ_1,τ_2) is αg^* -closed.

Proof:

Let $A\subseteq X$ be $\tau_i-g\alpha^*$ -closed set, then $cl_\alpha(A)\subseteq U$ where $A\subseteq U$ and U is $\tau_i-g\alpha$ -open, then U^c is $\tau_i-g\alpha$ -closed, hence U^c is $\tau_i-\alpha g$ -closed, then U is αg -open and $cl_\alpha(A)\subseteq U$, i.e. A is αg^* -closed.

Proposition (3.6)

Every αg^* -closed subset of a bitopological space (X,τ_1,τ_2) is gs^* -closed.

Proof:

Let $A\subseteq X$ be $\tau_i-\alpha g^*$ -closed set, then $\alpha cl_{\tau_{i=1,2}}(A)\subseteq U$ where $A\subseteq U$ and U is $\tau_i-\alpha g$ -open set ,i=1or2 , since $\alpha cl_{\tau_i}(A)=A\cup cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))$,i=1,2 , then $cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))\subseteq \alpha cl_{\tau_i}(A)\subseteq U$,i=1,2, but $\text{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq cl_{\tau_i}(\text{int}_{\tau_i}(cl_{\tau_i}(A)))$,i=1,2 then $\text{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq U$,i=1,2. Since $scl_{\tau_{i=1,2}}(A)$ is the smallest $semi$ -closed set containing A, so, $scl_{\tau_i}(A)=A\cup\text{int}_{\tau_i}(cl_{\tau_i}(A))\subseteq U$, i=1,2, and since U is $\tau_i-\alpha g$ -open, then U^c τ_i- is αg -closed, hence U^c is τ_i-gs -closed, then U is τ_i-gs -open and $scl_{\tau_{i=1,2}}(A)\subseteq U$, i.e. A is gs^* -closed.

Now, we will give examples of our new concepts.

Example (3.7)

Let τ_1 be the usual topological space and $\tau_2=\{X,\phi\}$, let $U=(a,b)$ be an open interval, then U is τ_1-sg -closed [], now let $A=(c,d)$ such that $a<c<d<b$, since $cl(A)=[c,d]$, so $\text{int}_{\tau_1}(cl_{\tau_1}(A))=(c,d)$, and since

$Scl_{\tau_1}(A) = A \cup \text{int}_{\tau_1}(cl_{\tau_1}(A))$
$\quad = \{c,d\} \cup \{c,d\}$
$\quad = \{c,d\}$
$\quad \subseteq \{a,b\}$
i.e. $A = \{c,d\}$ is sg^* -closed.

Example (3.8)

Consider the above example, and by proposition (3.3) we have $A = \{c,d\}$ is also gs^* -closed.

Example (3.9)

Consider the example (2.18), we have the set $\{b,c\}$ is α -closed, say U where $X = \{a,b,c,d\}$ and $\tau_1 = \{X,\phi,\{a\},\{c\},\{a,c\},\{a,b,d\}\}$, $\tau_2 = \{X,\phi,\{a,c\}\}$ so by proposition (2.9) we have $U = \{b,c\}$ is $\tau_1 - g\alpha$ -closed. Now let $A = \{c\}$, so $cl_{\tau_1}(A) = \{c\}$, $\text{int}_{\tau_1}(cl_{\tau_1}(A)) = \{c\}$ and $cl(\text{int}(cl(A))) = \{c\}$, but $cl_\alpha(A) = A \cup cl(\text{int}(cl(A))) = \{c\} \subseteq U$ i.e. $A = \{b,c\}$ is $g\alpha^*$ -closed set.

Example (3.10)

Consider the above example, and by proposition (3.5) we have $A = \{b,c\}$ is also αg^* -closed.

Definition (3.11) Let (X,τ_1,τ_2), (Y,ρ_1,ρ_2) are two Bitopological space ,A function $f : X \to Y$ is called:

1. gs^* -continuous if the inverse image of every ρ_i g -closed is τ_i gs^* -closed ,i=1 or2.
2. sg^* -continuous if the inverse image of every ρ_i g -closed is τ_i sg^* -closed ,i=1 or2.
3. $g\alpha^*$ -continuous if the inverse image of every ρ_i α -closed is τ_i $g\alpha^*$ -closed ,i=1 or2.
3. αg^* -continuous if the inverse image of every ρ_i α -closed is τ_i αg^* -closed ,i=1 or2.

Remark (3.12)

The relationships between the concepts in definition (3.11) summarized in the following diagram:

EMBED Equation.3 -continuous
EMBED Equation.3 -continuous
EMBED Equation.3 -continuous
EMBED Equation.3 -continuous

Diagram (3-2)

Now, we will prove every pointed in the above diagram in the following propositions:

Proposition (3.13)

Every sg^* - continuous function is gs^* - continuous function.

Proof:

Let (X,τ_1,τ_2), (Y,ρ_1,ρ_2) are two Bitopological space define $f:X\to Y$ which is sg^* - continuous function, and let A be a $\rho_i\ g$-closed in Y, since f is sg^* -continuous, then by definition (3.11.2), we have $f^{-1}(A)$ is $\tau_i\ sg^*$ -closed, and by proposition (3.3) $f^{-1}(A)$ is gs^* -closed, i.e f is gs^* -continuous function.

Proposition (3.14)

Every $g\alpha^*$ - continuous function is αg^* - continuous function.

Proof:

Let (X,τ_1,τ_2), (Y,ρ_1,ρ_2) are two Bitopological space define $f:X\to Y$ which is $g\alpha^*$ - continuous function, and let A be a α -closed in Y, since f is $\rho_i\ g\alpha^*$ -continuous, then by definition (3.11.3), we have $f^{-1}(A)$ is $\tau_i\ g\alpha^*$ -closed, and by proposition (3.5) $f^{-1}(A)$ is $\tau_i\ \alpha g^*$ -closed, i.e f is αg^* -continuous function.

Proposition (3.15)

Let $f:X\to Y$ and $h:Y\to Z$ are two gs^* -continuous functions then $h\circ f:X\to Z$ is also gs^* - continuous function, if every gs^* -closed in Y is g -closed

Proof:

Let (X,τ_1,τ_2), (Y,τ_1,τ_2), (Z,ρ_1,ρ_2) and $f:X\to Y$ and $h:Y\to Z$ are two gs^* -continuous functions, and let A be $\rho_i\ g$ -closed in Z, h is gs^* -continuous function then $h^{-1}(A)$ is $k_i\ gs^*$ -closed in Y, so by hypothesis $h^{-1}(A)$ is $k_i\ g$ -closed in Y, since f is gs^* -continuous function then $f^{-1}(h^{-1}(A))=(f^{-1}\circ h^{-1})(A)=(h\circ f)^{-1}(A)$ is $\tau_i\ gs^*$ -closed, i.e. $h\circ f$ is gs^* -continuous function.

Proposition (3.16)

Let $f:X\to Y$ and $h:Y\to Z$ are two sg^* -continuous functions then $h\circ f:X\to Z$ is also sg^* - continuous function, if every $k_i\ sg^*$ -closed in Y is $k_i\ g$ -closed

Proof:

Let (X,τ_1,τ_2), (Y,τ_1,τ_2), (Z,ρ_1,ρ_2) and $f:X\to Y$ and $h:Y\to Z$ are two sg^* -continuous functions, and let A be $\rho_i\ g$ -closed in Z, h is sg^* -continuous function then $h^{-1}(A)$ is $k_i\ sg^*$ -closed in Y, so by hypothesis $h^{-1}(A)$ is $k_i\ g$ -closed in Y, since f is sg^* -continuous function then $(h\circ f)^{-1}(A)$ is sg^* -closed, i.e. $h\circ f$ is $\tau_i\ sg^*$ -continuous function.

Proposition (3.17)

Let $f: X \to Y$ and $h: Y \to Z$ are two $g\alpha^*$-continuous functions then $h \circ f: X \to Z$ is also $g\alpha^*$ - continuous function, if every $g\alpha^*$-closed in Y is α-closed

Proof:

Let (X, τ_1, τ_2), (Y, τ_1, τ_2), (Z, ρ_1, ρ_2) and $f: X \to Y$ and $h: Y \to Z$ are two $g\alpha^*$-continuous functions, and let A be $k_i\ \alpha$-closed in Z, h is $g\alpha^*$-continuous function then $h^{-1}(A)$ is $k_i\ g\alpha^*$-closed in Y, so by hypothesis $h^{-1}(A)$ is $k_i\ \alpha$-closed in Y, since f is $g\alpha^*$-continuous function then $(h \circ f)^{-1}(A)$ is $g\alpha^*$-closed, i.e. $h \circ f$ is $\tau_i\ g\alpha^*$-continuous function.

Proposition (3.18)

Let $f: X \to Y$ and $h: Y \to Z$ are two αg^*-continuous functions then $h \circ f: X \to Z$ is also αg^* - continuous function, if every αg^*-closed in Y is α-closed

Proof:

Let $f: X \to Y$ and $h: Y \to Z$ are two αg^*-continuous functions, and let A be $\rho_i\ \alpha$-closed in Z, h is αg^*-continuous function then $h^{-1}(A)$ is $k_i\ \alpha g^*$-closed in Y, so by hypothesis $h^{-1}(A)$ is $k_i\ \alpha$-closed in Y, since f is αg^*-continuous function then $(h \circ f)^{-1}(A)$ is αg^*-closed, i.e. $h \circ f$ is $\tau_i\ \alpha g^*$-continuous function.

References .4

[1] J.C.Kelly ,"Bitopological spaces",Proc.London ,Math.Soc.13(1963)71-89.

[2] H. Maki, K. Balachandran and R. Devi, *"Associated topologies of generalized α -closed sets and α -generalized closed sets"*, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 15 (1994), 51-63.

[3] H. Maki, R. Devi and K. Balachandran, *"Generalized α -closed sets in topology"*, Bull. Fukuoka Univ. Ed. Part III, 42 (1993), 13-21.

[4] N. Biswas, *"On Characterization of Semi-Continuous Function"*, Atti. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., 48(8) (1970), 399-402.

[5] N. Falah," Certain Types of Generalized Closed Sets " ,Journal of AL-Nahrain university , To be published .

[6] N. Levine, *"Generalized closed sets in topological spaces"*, Rend. Circ. Mat. Palermo 19 (1970), 89-96.

[7] P. Bhattacharya and B. K. Lahiri, *"Semi-generalized closed sets in topology"*, Indian J. Math., 29 (1987), 375-382.

[8] S. Arya and T. Nour, *"Characterizations of s-normal spaces"*, Indian J. Pure Appl. Math., 21 (1990), 717-719.